The Spread of the Mathematical Method in Eighteenth-Century Germany: A Quantitative Investigation

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Abstract

In the eighteenth century, the German mathematician and philosopher Christian Wolff famously claimed that all sciences should apply the so-called *mathematical method*. Interpreters (e.g. Frängsmyr 1975; Friedman 1992) typically identify Wolff's mathematical method with the traditional axiomatic ideal of science, i.e. the tenet that a proper science should have an axiomatic structure. In this paper we argue against this identification. We show that several eighteenth-century authors who did reject Wolff's mathematical method in science did so while retaining the axiomatic ideal of science - which demonstrates that the two should not be identified. We argue that in the eighteenth century the expression 'the mathematical method' was ambiguous: it denoted both a broad axiomatic conception of science accepted by many authors as well as a specific take on the traditional axiomatic ideal of science. It is this specific take that was targeted by critics, we maintain, not the axiomatic ideal of science *tout court*, which was widely maintained. In order to substantiate our claims, we rely on information from an unusually large corpus of 164 eighteenth-century books on logic and philosophy in German and Latin, selected and annotated for the occasion using a substantially improved and enlarged version of the mixed (qualitative, quantitative and computational) method first developed in Betti et al (2019). We maintain that wide-scope historical-interpretive claims should rely on a corpus which is as large as possible, and employ precisely defined annotation schemes to capture differences and similarities between various conceptions in a way which is as accountable as possible. Our results should be understood as part of a longer-term ambition of making historico-interpretive investigations more scientific, i.e., controlled, explicit, and as objective as possible. [271 words]

1. Introduction

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It is known that Christian Wolff's views on the so-called mathematical method exerted a profound influence on eighteenth-century German thought (Frängsmyr 1975). There have been numerous qualitative studies on Wolff's mathematical method and the debates surrounding this method, including Lambert's and Kant's critique of the mathematical method (Tonelli 1956, Blok 2016, Shabel 2003, pp. 49-52, Dunlop 2009, 2013, Gava 2018. Heis 2014, Lawyine 2010, van den Berg (forthcoming)). However, to date there is no quantitative study into the popularity and spread of Wolff's mathematical method in eighteenth-century Germany. As a result, the precise impact of Wolff's mathematical method on eighteenth-century thought remains unknown. In this paper, we undertake a quantitative investigation of the popularity and spread of Wolff's mathematical method from 1720-1789 in German logic and philosophy. Methodologically, we employ a method that improves and expands on the novel mixed method (qualitative, quantitative and computational) introduced by Betti et al (2019): we use (qualitative) model-based annotation schemes to quantitatively capture conceptual differences and similarities between different conceptions in a way that is as accountable as possible. (see for computational and quantitative investigations inspired by the work of Betti's group on the so-called 'model approach', Sangiacomo 2019, Sangiacomo & Beers 2020, Bonino et al 2020, Žuradzki & Wiśniowska 2020. See for other computational and quantitative work in philosophy, which does not use models, Overton 2013, Mizrahi 2020a, 2020b, forthcoming).[2] In these studies, models function as qualitative frameworks that guide the computational and quantitative investigation of the corpus.

Our approach enables us not only to obtain a quantitative measure of the popularity and spread of Wolff's mathematical method in eighteenth-century Germany, but also to resolve some rather common interpretive confusions that surround the concept of Wolff's 'mathematical method'. In particular, interpreters (e.g. Frängsmyr 1975; Friedman 1992) typically identify Wolff's mathematical method with the traditional axiomatic ideal of science, i.e. the tenet that a proper science should have an axiomatic structure. In this paper we argue against this identification. We show that several eighteenth-century authors who did reject Wolff's mathematical method in science did so while retaining the axiomatic ideal of science - which demonstrates that the two should not be identified. We argue that in the eighteenth century the expression 'the mathematical method' was ambiguous. It denoted both a broad axiomatic conception of science accepted by many authors as well as a specific take on the traditional axiomatic ideal of science. It is this specific take that was targeted by critics, we maintain, not the axiomatic ideal of science *tout court*. We support this interpretation by showing that many critics of Wolff's specific take on the traditional axiomatic ideal of science nevertheless accepted the traditional ideal of axiomatic science, as modelled through de Jong &

[2] The model approach has been successfully applied in quantitative and computational investigations of the history of concepts in various domains and epochs (Sangiacomo 2019, Betti et al 2019, Sangiacomo & Beers 2020, Bonino et al 2020, Žuradzki & Wiśniowska 2020, Ginammi et al 2021; the latter relies on two variants of the Classical Model of Science, the model of axiomatic science we also adopt in this paper).

Betti's Classical Model of Science (2010). Insofar as our article contains quantitative data on the acceptance of de Jong & Betti's model, it is also the first quantitative study that demonstrates through data the widespread influence of this model.

We proceed as follows. In section 2 we introduce Wolff's mathematical method and the Classical Model of Science, and discuss influential interpretations of these topics. Section 3 contains descriptions of our corpus building method and annotation method. Section 4 provides an analysis of the quantitative results of our annotations. In section 5, finally, we move to a qualitative analysis of authors based on our annotations, illustrating that the axiomatic conception of science and Wolff's mathematical method should be distinguished.

2. The Mathematical Method and the Classical Model of Science

The use of the mathematical method in the sciences has a long history (Tonelli 1959). Frängsmyr states that, after having been championed through the centuries by Aristotle, St Thomas Aquinas and later scholasticism, Descartes, Arnauld and Nicole, Spinoza, von Tschirnhausen and Leibniz (Frängsmyr 1975, pp. 656-657), in eighteenth-century Germany the mathematical method was strongly associated with the work of Christian Wolff, who popularised it. It is known that Wolff was famous for claiming that this method should apply to all proper sciences. It is however doubtful whether Wolff articulated a unitary conception of the mathematical method. For one thing, as several commentators have noted (Frängsmyr 1975, p. 655; Tutor 2018, pp. 81-88; Cantù 2018, p. 372), Wolff used different terms to characterize the mathematical method, such as the 'mathematical', 'geometrical', 'demonstrative' or 'scientific' method. Moreover, Tutor (2018) describes various different characterizations of the mathematical method in Wolff's extensive *oeuvre*. In the following two paragraphs, we will, after Tutor, describe two different characterizations of the mathematical method by Wolff: one more specific and detailed (we call this *narrow*), and the other more general and generic (we call this *wide*).

Narrow. One of the most specific and detailed descriptions of the mathematical method is given by Wolff in his *Die Anfangsgründe aller Mathematischen Wissenschaften* (1710), in the chapter entitled *Kurzer Unterricht von der Mathematischen Methode oder Lehrart*.[3] Here, Wolff defines the mathematical method as the order which mathematicians use in the presentation of a science, which starts with definitions, proceeds to the axioms (*Grundsätzen*), and then proceeds to the theorems and problems (Wolff 1750 [1999], 5). According to Wolff, definitions ground the axioms and postulates of the mathematical sciences. The axioms and postulates, in turn, ground the theorems and problems of mathematics (ibid). Wolff distinguishes *nominal* from *real* definitions. Nominal definitions provide marks which allow us to know a thing and distinguish it from other things (Wolff 1750 [1999], p.6). For example, a definition of a circle in terms of the marks of a figure that has four equal sides and equal angles is a nominal definition. Real

[3] In this paragraph, we follow the account of Wolff's mathematical method given in van den Berg (forthcoming). For a more elaborate account of Wolff's mathematical method, see Blok 2016, Shabel 2003, pp. 49-52, Dunlop 2013, and Gava 2018. We discuss some of these authors in more detail below.

definitions show how the object of the defined concept is possible (ibid.). For example, Wolff provides a real definition of a circle by specifying the constructive procedure of moving a straight line around a point. The axioms and postulates of mathematics follow from definitions (either nominal or real) (Wolff 1750 [1999], p.16). For example, from the real definition of a circle specified above we can immediately infer the axiom that the straight lines drawn from the center to the perimeter are equal to each other (ibid.). Because axioms and postulates immediately follow from definitions, they do not require proof. The truth of axioms and postulates is evident once we understand the definitions from which they follow (Wolff 1750 [1999], p.17). Finally, theorems are deduced from definitions, axioms and postulates through syllogistic inferences (ibid., p. 21-22). Hence, Wolff adopted a strictly axiomatic conception of science in which definitions ground axioms and postulates, and theorems are syllogistically derived from the axioms and postulates.

Wide. A more generic characterization of the mathematical method is given by Wolff in his *Ausführliche Nachricht* of 1726 (see also Tutor 2018). There, Wolff simply states that the mathematical method requires us to define all words, to prove all propositions, and to order all definitions and propositions in an axiomatic fashion. As Wolff articulates the point:

> In my presentation of the matters I have mainly focused on three things, 1. That I did not use a word that I did not define, where otherwise an ambiguity could arise through the use of the word, or a ground for the proof was missing: 2, that I did not admit a proposition, and used it in the following as a premise in inferences to prove other propositions, which I had not proven beforehand: 3, that I constantly combined the subsequent definitions and propositions with one another and derived them from one another in a continuous fashion (Wolff 1973 [1733], pp. 52-53. Our translation).[4]

Thus, in this generic description, the mathematical method is simply understood as an axiomatically ordered science with a deductive structure. This generic description differs from the more specific description given in *Die Anfangsgründe aller Mathematischen Wissenschaften* (1710), for in the generic description we do not explicitly find the requirement that axioms and postulates are derived from definitions. Wolff's generic description of the mathematical method was common in the eighteenth century. For example, in the lemma on the mathematical method of Zedler's *Grosses vollständiges Universal-Lexicon aller Wissenschaften und Künste* (1739), the mathematical method is described as a method which (i) defines all words that need a definition, (ii) proves all propositions that require a proof, (iii) and places definitions and

[4] In meinem Vortrage der Sachen habe ich hauptsächlich auf dreyerley gesehen, 1. daß ich kein Wort brauchte, welches ich nicht erkläret hätte, wo durch den Gebrauch des Wortes sonst eine Zweydeutigkeit entstehen könte, oder es an einem Grunde des Beweises fehlete: 2, daß ich keinen Satz einräumete, und im folgenden als einen Förder-Satz in Schlüssen zum Beweise anderer brauchte, den ich nicht vorher erwiesen hätte: 3. daß ich die folgende Erklärungen und Sätze mit einander beständig verknüpffte und in einer steten Verknüpffung aus einander herleitete. (Wolff 1973 [1733], pp. 52-53).

propositions in their proper order (Zedler 1739, p. 2053). Here, again, it is not explicitly stated that axioms and postulates are grounded by definitions.

We take Wolff's generic conception of the mathematical method to be aptly modelled by the Classical Model of Science (hereafter: the Model) developed by Willem de Jong and Arianna Betti (2010). The Classical Model of Science is an interpretative tool that describes a traditional axiomatic ideal of science accepted by many philosophers and scientists (Betti and van den Berg 2014). According to de Jong & Betti, the Model, which can be traced to Aristotle, was influential for more than two millennia and was adopted among others by Descartes, Pascal, Newton, Leibniz, Wolff and Kant, and later by Bolzano, Husserl, Frege and Leśniewski. According to the Model, a proper science is a system *S* that satisfies the following conditions:

(1) All propositions and all concepts (or terms) of *S* concern a *specific set of objects* or are about a *certain domain of being*(*s*).
(2a) There are in *S* a number of so-called *fundamental concepts* (or terms).
(2b) All other concepts (or terms) occurring in *S* are *composed of* (or are *definable from*) these fundamental concepts (or terms).
(3a) There are in *S* a number of so-called *fundamental propositions*.
(3b) All other propositions of *S follow from* or *are grounded in* (or *are provable* or *demonstrable from*) these fundamental propositions.
(4) All propositions of *S* are *true*.
(5) All propositions of *S* are *universal* and *necessary* in some sense or another.
(6) All propositions of *S* are *known to be true*. A non-fundamental proposition is known to be true through its *proof* in *S*.
(7) All concepts or terms of *S* are *adequately known*. A non-fundamental concept is adequately known through its composition (or definition). (De Jong & Betti 2010, p. 186)

It may be thought that the Model merely systematizes the axiomatic ideal of science that was popularized through Wolff's mathematical method. If one adopts this reading, the Model and Wolff's mathematical method are identical. Indeed, as we have seen, there is some evidence for equating the two. Wolff's generic conception of the mathematical method, also articulated in Zedler's lexicon, closely resembles the Model. The requirement to define all words that need a definition corresponds to conditions (2a) and (2b) of the Model, whereas the requirement to prove all propositions that require a proof corresponds to conditions (3a) and (3b) of the Model. Can we then conclude that the Model is identical to Wolff's mathematical method?

In the secondary literature, there is a tendency to do just that, i.e. equate the mathematical method with a traditional axiomatic ideal of science, namely the tenet that a proper science should have an axiomatic structure. For example, Frängsmyr describes the mathematical method as follows:

> Scientific method described the definite rules in accordance with which the philosopher should work. Slightly simplified, this meant that one was to obey the laws of logic and, in

> particular, the laws of deduction. Wolff pointed out his meaning more exactly in some passages: no principles may be used if they are not fully proved, and no new principles are accepted unless they are derived from such principles; one is not allowed to diverge from the meaning that words generally have, and if new words and concepts are introduced they must be properly defined; things and phenomena which are of a different nature must be given different names, and philosophical terms once accepted are not to be changed. Starting from axioms, clear definitions, and distinctions, and following these rules, the philosopher should be able to connect the truths with each other by the deductive method and arrive at incontrovertible conclusions. In this way, he would obtain just as reliable results in philosophy as in mathematics. (Frängsmyr 1975, p. 656).

Frängsmyr's description of the mathematical method resembles Wolff's generic description of this method and conforms to the Model. Frängsmyr identifies the mathematical method with a science obeying the laws of deduction, and notes that all propositions of a science should be proved (corresponding to conditions (3a) and (3a) of the Model), notes that new concepts must always be defined (corresponding to conditions (2a) and (2b) of the model), and notes that the mathematical method guarantees the certainty of a science (corresponding to condition (6) of the Model). Similarly, Michael Friedman, describing the application of the mathematical method to metaphysics in Wolff, gives the following description of Wolff's method:

> On the one hand, the Wolffian philosophy had attempted to develop metaphysics in the form of a strictly deductive system, *more geometrico*: metaphysics was to begin with the most abstract and general concepts (such as being, essence, attribute, and the like) and to construct a chain of definitions of successively more concrete and particular concepts (such as body, state, motion, and the like). At the same time, one began with the most abstract and general principles (the highest of these being the principle of contradiction) and built up a chain of syllogisms leading from there to the most concrete principles: at this point the entire structure could be confronted with experience. (Friedman 1992, p. 15)

Like Frängsmyr, Friedman provides a description of the mathematical method that resembles Wolff's generic description of this method. Friedman notes that metaphysics must start from fundamental concepts and define non-fundamental concepts in terms of these fundamental concepts (corresponding to conditions (2a) and (2b) of the Model), and that metaphysics is based on fundamental principles from which we syllogistically derive non-fundamental propositions (corresponding to conditions (3a) and (3b) of the Model). Both Frängsmyr and Friedman thus provide a reading of the mathematical method that supports identifying this method to the Model.

However, the identification of the Model with the mathematical method raises interpretative problems. De Jong and van den Berg have argued that Kant follows the Model (de Jong 2010, van den Berg 2011, van den Berg 2014, Chap. 2). However, Kant famously rejects Wolff's mathematical method in several of his philosophical writings. This means that if the mathematical method and the Classical Model are identical, Kant would also have had to reject the Classical Model, which is inconsistent with De Jong and Van den Berg's findings. So, are these findings then incorrect? Or is it rather, like Van den Berg (forthcoming) recently argues, that there is a way to make Kant's rejection of the mathematical method consistent with accepting the Classical Model of Science? Van den Berg's suggestion is that it is wrong to identify the Model with Wolff's mathematical method.

In this paper we side with van den Berg's stance, and we provide new supporting evidence for it from a large-scale quantitative analysis of the spread of the mathematical method in eighteenth-century Germany. On the basis of our quantitative analysis, we highlight a number of authors (insert percentage) who were critical of Wolff's mathematical method but accepted conditions (2) and (3) of the axiomatic ideal of science codified by the Model, the two conditions which capture the core of the structure of an axiomatic science. We show that it was a common position in the eighteenth-century to be critical of the mathematical method while accepting a traditional axiomatic ideal of science as codified by the Model. We can also express the point by stating that Wolff's generic description of the mathematical method, as articulated by Michael Friedman in terms of conditions (2) and (3) of the Model, was accepted by most authors in the eighteenth century. The focus of critique against Wolff was directed at various specifics of his position. There were, of course, important differences between eighteenth-century authors' conceptions of science and different critiques of the mathematical Method, which we discuss below and in section 5.

According to our reading, then, whilst Wolff's narrow conception of the mathematical method was criticised, his wide conception of his method - condition (2) and (3) of the Model - was widely accepted. We think it is important to stress this subtle point because if we ignore it we lose sight of important continuities in eighteenth-century philosophical thought. For example, in his insightful discussion of Kant's critique of the mathematical method, Friedman correctly stresses that in his pre-critical phase Kant espoused a Newtonian methodology in metaphysics, which starts from certain and uncontroversial phenomena and consisted in providing a "deduction from the phenomena" (Friedman 1992, 15-16). Friedman further correctly notes that Kant's criticism of Wolff's mathematical method implied that in metaphysics we cannot start with definitions as a basis for inferences (Friedman 1992, p. 21, 24), a topic which we will revisit later in the paper. However, Friedman concludes that according to Kant "metaphysics must renounce a strictly deductive or synthetic method in which all subsequent propositions are derived from a small number of initial principles, and must instead adopt a quasi-inductive or regressive method which takes as its basis a large number of heterogeneous unprovable propositions as data" (Friedman 1992, p. 22). This conclusion, we argue, is too strong, because Kant did follow Wolff in conceiving of (a future) metaphysics as a deductive science where non-fundamental propositions are derived from fundamental principles (condition (3) of the Model). Friedman's conclusion seems to be a result of identifying Wolff's mathematical method with his generic description of this method, and thinking that Kant rejects the latter. However, as we will show, Kant accepted this general conception of science as codified by the Model. Similarly, Sutherland provides a very insightful discussion of Kant's critique of the mathematical method, noting correctly that according to Kant "one should not attempt to imitate mathematical method in philosophy, above all by attempting to start one's investigation with definitions." (Sutherland 2010, p. 176). However, Sutherland interprets Kant's critique of the mathematical method, especially his critique of the attempt to start metaphysics with definitions, in such a manner that he concludes that "Kant is rejecting the entire Wolffian approach to philosophy" (Sutherland 2010, p. 177). Although Sutherland correctly identifies Kant's critique of the Wolffian account of the role of definitions in metaphysics, this statement, we argue, does not do justice to important continuities between Wolff and Kant, in particular their shared generic conception of proper science as described by the Model. By stressing the similarities between Wolff and Kant, our position is similar to that of Gava (2018). Gava argues that Kant's critique of the mathematical method makes us miss important continuities between Wolff and Kant, such as Kant's praise of Wolff's method, given in the Preface to the first Critique, where Kant states that in constructing a future metaphysics "we will have to follow the strict method of the famous Wolff" (Bxxxvi). As important elements of continuity, Gava mentions both these authors' views on systematic science, systematicity being a condition of proper science, and their views that philosophy is deductive (Gava 2018, p. 303). We add to Gava's argument that Wolff and Kant shared a generic conception of proper science, codified by the Model.

It is important to note that Wolff's mathematical method was also a position on the discovery of certain truths through experience. In his *Die Anfangsgründe aller Mathematischen Wissenschaften* (1710), as well as in his works on logic, Wolff provides comments on how experience can lead to definitions and how experience can be transformed into universal truth (Wolff 1750 [1999], p.8.). This role of experience has been stressed by Dunlop, who notes that Wolff, even though he accepts a traditional axiomatic conception of science, is not a traditional rationalist and assigns experience a role in demonstrating the possibility of concepts and providing definitions, even in mathematics (Dunlop 2018). Similarly, Dunlop (2013) argues for a broad interpretation of Wolff's mathematical method, one which stresses the role of perception and imagination in concept formation. In line with Dunlop's views, van den Berg and Demarest (2020), who also stress Wolff's axiomatic ideal of science, describe the role of experience in empirical science for Wolff, including the life sciences, where experience provides us with premisses on the basis of which we derive conclusions through strict deductive demonstrations (on this point, see also Anderson 2015). These specific aspects of the mathematical method are often different from Kant's conception of science. For example, Kant's account of proper natural science implies that a proper science must be based on synthetic a priori principles (alongside empirical principles), where a priori means, in contrast to Wolff, "justified independent of experience".[5] Kant's account of the a priori and his view on the grounding of proper natural

[5] Wolff takes a priori knowledge to be knowledge inferred from known knowledge, i.e, a priori knowledge is knowledge inferred on the basis of a valid syllogism. See Beck 1996, p. 265, Vanzo 2015, pp. 242-244, Favaretti Camposampiero 2016, p. 365, van den Berg & Demarest 2020, n5. Kant takes a priori knowledge to be knowledge justified independent of experience. On Kant's notion of a priori and its similarities to Lambert's notion of the a priori, see Watkins 2018.

science are certainly distinct from Wolff. However, once again we stress that it is also important to note the continuities between Wolff and Kant, such as their shared generic conception of proper science as codified by the Model, because it is within this shared framework that Wolff and Kant develop their own peculiar conception of proper science. In the following, we will take pains to stress both the continuities and the discontinuities between different authors' reflection on the mathematical method and science, allowing us to sketch discontinuities within continuities.

Finally, it is necessary to highlight the pervasive influence of the Model in eighteenth-century Germany since there are many present-day interpreters who reject the impact of this axiomatic idea of science on empirical science. Plaass argues that Kant could not have adopted an axiomatic conception of science, since empirical sciences are based on empirical principles and an axiomatic conception of science based on a priori principles would not need principles of experience (Plaass 1994, p. 235-236. The same point is made by Sturm 2009, p. 153.). In other words, it is not clear how empirical sciences fit an axiomatic conception of science. In a similar vein, Gava (2018, p. 292n28) states that empirical sciences cannot be deductive. This line of reasoning has been rejected by van den Berg and Demarest (2020) and van den Berg (2014, Chapter 2), which show that in Wolff and Kant empirical sciences based on empirical principles can constitute axiomatic sciences. Anderson's discussion of Wolff (2015, chapter 3) also stresses the importance of syllogistic or deductive inference in the empirical sciences. Indeed, Wolff, as van den Berg and Demarest (2020) stress, took the construction of an axiomatic physics based on empirical principles to be an important step in grounding physics as a proper science. In the present paper, we adopt a quantitative method that further substantiates the widespread acceptance of an axiomatic idea of science, codified by conditions (2) and (3) of the Model. However, we also point out the existence of a tradition of thought that rejects the *certainty* of the empirical sciences, arguing that empirical sciences can only yield probable knowledge (see section 5.2). We show that several authors in this tradition, while often accepting conditions (2) and (3) of the Model, and thus assigning to any science an axiomatic structure, rejected the knowledge postulate of the Model (condition (6)), according to which all propositions are *known to be true*. By highlighting this tradition, we describe discontinuities within eighteenth-century philosophical thought on science.

3. Corpus and Method

In this section, we describe our corpus building procedure (3.1) and our method for annotating the historical books contained in the corpus (3.2). Both the corpus building procedure and the method for annotating are highly complex, and comprise a large number of interrelated steps. We have exhaustively documented our methods and research practices. As we are in the process of writing a separate paper on the complete corpus building workflow we used (removed for blind review), and another separate paper on the complete annotations workflow we adopted (removed for blind review), we focus in this section on the essentials rather than on the details of the two methods, and provide links to the necessary data and documentation. In particular, full annotation results can be found in the Appendix.

3.1 The Expert Corpus and how we built it

Our analysis is based on a corpus of hundred-sixty books on logic and philosophy published in eighteenth-century Germany written by the following hundred-fifteen authors: [Doc Sheet Zotero]

Abel	Diez	**Heinicke**	**Lambert** (3)	Polzius	Storchenau (2)
Amort (3)	Dorsch	**Henning** (2)	Layritz	Reis	**Strähler**
Argens	Duhan	Heydenreich	Locherer	Reimarus	Tittel (2)
Baumgarten (3)	Ebeling	Hiller	Lossius (2)	Renz	**Unzerin**
Besecke	Eberhard	Hilliger	Ludovici (2)	Reuss	Vierthaler
Bilfinger (2)	Ebert	Hissman (2)	Mako	Roetenbeck	Vogl
Brucker (3)	Engel	Hofer	**Mangold** (2)	Rost	Vogt
Buches	Ernesti (2)	Hofmann (2)	Mayr	**Rüdiger**	**Walch**
Buddeus	**Feder** (4)	**Hoheisel**	Meier (2)	Gravesande	**Watts**
Burkhäuser	Ferber	Hollmann	Meiners	**Scherffer** (2)	Weber
Cartier	Flögel	Jacobi (4)	**Mendelssohn** (2)	Schlosser	**Weis**
Clemm	Frantz	Job	**Müller** (2)	Schnell	Weishaupt
Cordier	Garve	Justi	Neubauer	Schulze	Wenzl
Corvinus	Gordon	**Kant** (3)	Oberhauser	Schütz	Wietrowski
Crusius (2)	Gottsched	Kästner	Palaeocaenus	Selle	Will
Dalham	Gravesande	Knigge	Panger	Sendelbach	Wolff (11)
Darjes (2)	Gremner	Knutzen	Platner	Stattler	**Wüstemann**
Dedelley	Gufl	Koelbele	Plessing	**Steinbart**	Ziegler
Desing	Hagen	Kraus	Ploucquet (2)	Stolle (2)	Zopf
Dietler	Harris				

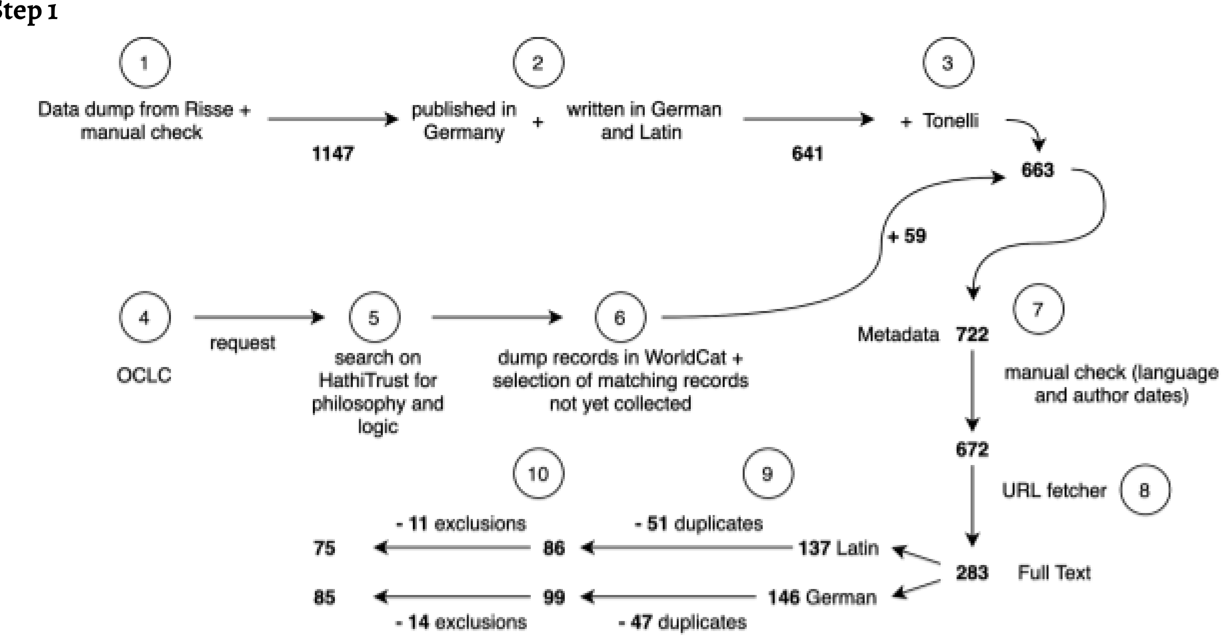
In bold we have marked the authors that, in our analysis, turned out (explicitly or implicitly) to accept conditions (2) and (3) of the Classical Model of Science but to reject Wolff's Mathematical Method (20/115, about 17,39%). This percentage includes both thinkers who rejected Wolff's Mathematical Method altogether, i. e. for *any* science ( ) or accept it exclusively for mathematics ( ). This is, at a glance, our main finding. Before we say more on this result, we describe the procedure we followed to reach it. Note that all authors without exception (100%) accept the Classical Model of Science.

How did we identify the sources forming our corpus and how did we analyse them? Following up on the recommendations of Betti et al (2019) on objective corpus building for wide-scope historical claims, we set out only to use a "properly delineated and well-delimited corpus serving as the explicit textual basis for the research". To this, we add a further methodological constraint, namely that not only should one indicate the exact corpus on which results are based, a well-grounded method should be used to identify (and possibly to collect) sources, a method that is to a large extent replicable, and explicitly aimed at reducing biases.

One such method is the *Universal Bibliography First* method described in Betti 2020a and 2020b, which takes as a starting point bibliographic metadata openly accessible in large-scale online repositories, first filtered by *subject*. In this paper we applied a version of this method.

We focused on sources in logic next to general philosophical sources because, as even a cursory manual inspection of logic books in the eighteenth-century reveals, these sources tend to contain explicit discussion of the method of proper sciences; we also presumed that philosophy books would often contain important reflections on the method of philosophy. We settled for books in both Latin and German because German-speaking authors like Wolff published in both languages; as known [here something about the relation Latin-German]. We decided to set the timespan from 1720 to 1789. The 1720s are the decade after which Christian Wolff published his influential *Die Anfangsgründe aller mathematischen Wissenschaften* (1710), and his German Logic (1713), both influential works in which he articulated his ideas on the mathematical method. We conjectured that by the 1720s the impact of Wolff's mathematical method must have been visible in printed sources, and chose 1720 as starting point. We chose 1789 as the end point because in the late 1780s Immanuel Kant became the dominant German philosopher (Rohlf 2020), and Kant explicitly rejected Wolff's mathematical method in his *Critique of Pure Reason* (1781/1787). Our workflow comprised four steps.

Step 1



Graph 1

We started by collecting all entries from Wilhelm Risse's *Bibliographia Logica* (1965-1979) for our period published in Europe (thousand and hundred forty-seven records); from these we only retained books that were from Germany and written in German or Latin. We construed 'from Germany' as 'published in a city that was German at the time' (i.e. belonged to the Holy Roman

Empire or Prussia); we also included Riga among the cities.[6] This left us with six-hundred forty-one records (55.8%). We then supplemented our list with twenty-two books in philosophy mentioned by Tonelli (1959), that were relevant to the mathematical method in our period, on count of the fact that [...]. The result was a list of six-hundred sixty-three records.

Step 2

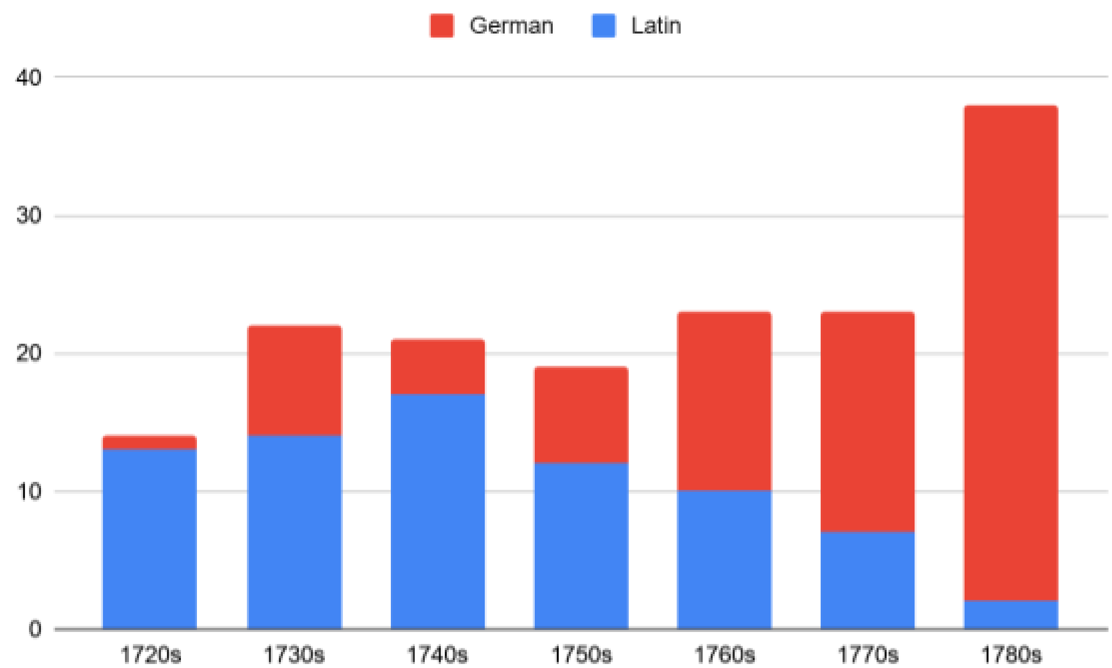
More records were added as follows. We searched HathiTrust[7] for books catalogued as *philosophy* or *logic* in the subject field and published between 1720 and 1789; we filtered manually the results according to our language and area requirements, then added any item that turned out to be missing on our Risse & Tonelli list from **Step 1** [722-633=x, add percentage of foundlings with respect to the 722 total]. We chose HathiTrust at this Step instead of WorldCat, Google Books, The Open Library or The Internet Archive because… This yielded a final dataset of seven-hundred twenty-two bibliographic records.

Step 3

Step 3 consisted in finding as many digitised texts from our list as was possible and feasible in a semi-automatic way in reasonable time. We built a URL fetcher in partnership with OCLC, which deploys a standard predictive algorithm to search for digitised texts on Google Books matching our list. The URL fetcher matched two-hundred eighty-three bibliographic metadata records to URL addresses of full texts in Google Books. We checked all three-hundred twelve matches to full texts manually and excluded fifty (e.g. full texts that turned out not to match language or time period criteria, including matches to editions of the text published outside our time period); we also excluded one hundred and one duplicates (identical URLs matched to multiple records). Our final list counted one hundred and sixty full texts (seventy-five in Latin and eighty-five in German), [insert percentage of 722] The annotators worked with a final corpus of 283 digitised full texts (137 Latin texts and 146 German texts), with associated bibliographic metadata records. 98 texts were excluded from the corpus because the URL link matched with these texts was identical to the URL link to other texts, therefore they were considered as "duplicates". Moreover, the annotators excluded from the corpus 25 texts because (i) they were not published in Germany, (ii) they were not published between 1720 and 1789, (iii) they were not written in Latin or German, (iv) or because the match between the bibliographic metadata and the record behind the URL link was incorrect and no correct URL link to the

[6] This needs explanation, since Riga in our period was part of the Russian Empire. [HEIN]
[7] https://www.hathitrust.org/

correct full text was found by the annotator. After these procedures, the final Latin[8] and German[9] corpora resulted in 160 texts (75 written in Latin and 85 written in German).[10]



Graph 2

Step 4

Step 4 consisted in annotating all one hundred and sixty full texts, following a method similar to the one used in Betti et al 2019, though more complex, due to different tasks and aims. in Betti et al 2019 presence or absence of a certain concept fixed in a model is annotated in authors and texts previously unknown to the interpreters.[11] The main difference is that in Betti et al 2019 one

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[8] Latin Corpus Philosophical Analysis (pdfs).

[9] German Corpus for Philosophical Analysis (pdfs).

[10] The bibliographic metadata of the texts are reported in Latin corpus metadata for Philosophical Analysis and German corpus metadata for Philosophical Analysis. Each document contain two worksheets: the worksheet *Final (with exclusions and duplicates)* contains also metadata of excluded texts and duplicates, while the worksheet F*inal (without exclusions and duplicates)* contains the metadata of the final corpus of texts published between 1720 and 1789 in Germany.

[11] [CAN GO TO CONCLUSION AND FUTURE WORK] The three steps of the method are a more basic, rougher and manual version of Steps 1, 2, 3 and 5 of Betti et al 2020, in which a semiautomated expert workflow is fixed for annotating concepts as part of a method to construct ground truths for computational evaluation, aided by software called HitPaRank. The main difference with the full workflow of Betti et al 2020 is that our corpus consists of (unsegmented) pdf files with a text layer provided by Google Books. This means that the units of texts we annotate here are 'passages', a vague unit

specific term ('conceptual scheme') in one language (English) is searched for, and then assessed by annotators - by interpreting the term's context - as expressing or not a certain previously modelled concept(ion). By contrast, in the present work we had to figure out all possible terms in two languages (Latin and German) that could be taken to express multiple key concepts appearing in previously modelled concept(ion)s, then search for these terms, thus establishing a technical glossary for general history and philosophy of science for the period, iteratively and along the way. We then annotated the terms from the glossary in the sources. We document the method at Step 4 in detail in the next subsection.

3.2 The Annotation Method

The hypothesis we evaluate in this paper is whether in our corpus we can identify sources who rejected Wolff's specific mathematical method while endorsing the axiomatic ideal of science, and if so, the spread and popularity of the first with respect to the second. Our annotation method comprised three steps. *Annotating* at Step (iii) consisted in jotting down for each book in our corpus whether we found textual evidence for or against adherence to Wolff's Mathematical Method (MM) or to the Classical Model of Science (CMS) through detecting the presence of terms or phrases (textual evidence) that express that conception, and jotting down those terms or phrases. In practice, this means making a note on a separate data sheet in form of a reply to a number of questions regarding relevant concept-term mappings. Here is an example:

[EXAMPLE]

The textual evidence that is *relevant* to annotate, i.e. which terms signal the concep(tion)s of interest, is fixed at Step (i-ii). At Step (i) (*Modelling*) both conceptions (MM and CMS) are *modelled* in an explicit, shareable and rather precise way. By 'modeling' (Betti & van den Berg 2014, Betti & van den Berg 2016) we mean associating to each of the two concept(ion)s - MM and CMS - a network of logically related (sub)concepts that we ourselves establish as a characterisation or definition of that concept(ion); the conceptual network gives necessary and sufficient *conditions* for it, such that finding evidence for the network in text means finding evidence for the conception in text. Step (ii) (*Term/Concept Mapping*) consists in an iterative construction of a technical glossary of terms in the languages of interest for each concept in the network composing the model. The construction starts from some initial terms known to be used by known authors of the period through expert knowledge and tentative translations that might or might now be actually found in other texts (e.g. LATIN WORD and GERMAN WORD

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which is delimited in a replicable and automatic way. In annotating the passages, annotators have carried out term expansions at Step 3 manually and 'on the fly' (given that HitPaRank is inapplicable to the corpus at it stands). Further, annotation is performed by only one annotator per language, so no two passages have been annotated by different people. Finally, in this paper our aim has not been to maximize recall, but we did not endeavour to find all the passages relevant to a certain question in the corpus.

for *primitive* or *fundamental terms*).[12]
[SMALL SUMMARY OF WHAT HAPPENS IN THE EXAMPLE] apply in any such book, then they adhere to the CMS, so we annotate whether the conditions apply for each.

Step (i). We needed to model two conceptions: the axiomatic conception of science in general and Wolff's mathematical method as a specific take on it. For the former, we adopt an already existing and well-tested model, the Classical Model of Science (CMS), which we introduced in section 2. We saw that the CMS comprises seven conditions, but for our purposes we only need to consider conditions 2 and 3:

In a proper science as a system *S*:

2a. There are in *S* a number of so-called fundamental concepts (or terms).
2b. All other concepts (or terms) occurring in *S* are composed of (or are definable from) these fundamental concepts (or terms).
3a. There are in *S* a number of so-called fundamental propositions.
3b. All other propositions of *S* follow from or are grounded in (or are provable or demonstrable from) these fundamental propositions.

As Betti & van den Berg 2014 specify, the (categorematic) terms appearing in 2-3, like *fundamental*, *definable* or *concept* stand for abstract concepts that are determinables. They are just labels indicating a certain concept, but they are by no means the actual lexical entries we find in our corpus. For one thing, the labels are in English simply because of the vehicular function of the language, but our corpus contains only Latin or German terms; furthermore, annotators often do not find literal translations of the labels appearing in a model, but context-dependent functional equivalents of them. Take *fundamental proposition*: the Latin annotator marks, aided by context, phrases such as *axioma*, *propositio fundamentalis* and *propositio primitiva* as functional synonyms for *fundamental proposition,* while the German annotator [INSERT GERMAN].

The model we constructed for (endorsement of) Wolff's mathematical method and conditions (2) and (3) of the Classical Model looks as follows:

The Mathematical Method (Wolff's mathematical method):

[12] As Ginammi et al 2021 stress, this step is not only mandatory in quantitative and computational investigations, but also in traditional ones: the difference is that in traditional studies concept-term mappings are routinely kept implicit by interpreters, while in quantitative and computational ones, like the present, those mappings have to be made fully explicit.

A. The Mathematical Method should be endorsed.[13]

B. The Mathematical Method is universal, i.e., it should be applied to all sciences, including philosophy, or it should be applied to at least another science.

C. Definitions ground axioms and/or postulates (at least for the sciences for which B holds).

We operationalised the Model we have by constructing an annotation scheme for it. This annotation scheme guided the annotation of historical books contained in the corpus. Below, we show the annotation scheme of the Model's condition A./B./C.

Annotation Scheme for Conditions A./B./C.

A. The text contains an endorsement of the mathematical method, i.e., a sentence that literally says (modulo synonymy) that we should follow the mathematical method. That is, there should be a sentence containing the German trigram 'die mathematische Lehrart' or 'Wolff's mathematical method' and a verb of endorsement or the Latin bigram 'Methodus/Methodo Mathematica'.

A-Questions

A-Q1 What terms of endorsement are used?

A-Q2 Which words corresponding to "Mathematical Method" does the passage contain?

A-Q3 Is there reference to Wolff?

A-Q4 Is there any other information that influenced your decision on criterion A?

B. The text claims that the mathematical method is universal, i.e. it should be applied to all sciences, including philosophy, or it should be applied to at least another science.

B-Questions

B-Q1 What terms or expressions relating to universality are used?

B-Q2 Is the field of philosophy explicitly mentioned?

B-Q3 Are other sciences explicitly mentioned?

B-Q4 If yes, what sciences are explicitly mentioned?

B-Q5 Is there any other information that influenced your decision on criterion B?

C. The text claims that definitions ground axioms and/or postulates (at least for the sciences for which B holds).

[13] When using the Classical Model of Science, we did not use a condition that states that the Classical Model should be endorsed because, as de Jong & Betti (2010) argue, the Classical Model is an *Ideengeschichtlich* historical tool that *reconstructs* an often implicit conception of science held by many authors. As a historical reconstruction, the Classical Model was unknown to the historical authors we studied. The situation is different for Wolff's mathematical method, which was widely known to authors in the eighteenth century, and of which many authors explicitly said that it should be followed.

C-Questions

C-Q1 Which words or expressions corresponding to grounding are used?
C-Q2 Which words or expressions corresponding to axioms are used?
C-Q3 Which words or expressions corresponding to postulates are used?
C-Q4 Was it 'and' or 'or' (both axioms and postulates, or only one)?
C-Q5 Is there any other information that influenced your decision on criterion C?

We score each condition as follows.

	-1	-0,5	0	+0,5	+1
A.	explicit rejection	implicit rejection	no info	implicit endorsement	explicit endorsement
B.	(A) is at most -0,5 and explicit denial of applicability to even one science	(A) is at most -0,5 and implicit denial of applicability to one (or more) specific science	no info	(A) has at least 0,5 and the mathematical method should be applied to at least a particular science	if (A) has at least 0,5 and the mathematical method should be applied to all sciences
C.	(B) is at most -0,5 and explicit denial of definitions grounding axioms and postulates	(B) is at most -0,5 and implicit denial of applicability to one (or more) specific science	no info	(B) has at least 0,5 and there is implicit endorsement definitions ground axioms and postulates for the relevant sciences (one or more)	(B) has at least 0,5 and there is explicit endorsement definitions ground axioms and postulates for the relevant sciences (one or more)

The annotation scheme for conditions (2a) and (2b) for the Classical Model of Science runs as follows:

Annotation Scheme for Conditions 2a./2b.

2a. The text contains an endorsement of (2a), that is, the text contains one or more sentences that literally say (modulo synonymy) that a science contains *Grundbegriffe*, *erste Begriffe, Elementarbegriffe, höchste begriffe* or *einfache Begriffe*, or in Latin (*Primum*) *Definitiones/Notiones*, *Unum*, *Fundamentum Esse* (explicit endorsement); alternatively, the text contains one or more sentences that literally say that a science contains concepts that are more fundamental than other concepts (*einfacher*, *höher, zusammengesetzt, nicht elementar*; *Ideae, quae definitio includit*; *ideae, definitionibus continentur*) (implicit endorsement). NB: the text might contain hyponyms or hypernyms of *Begriffe*! For instance: *rationata*, *consecutiva*, *definiens* etc.

2a-Questions

2a-Q1 Which words or expressions corresponding to fundamental concepts are used?

2a-Q2 Which terms or expressions specify the relation between fundamental concepts and science?

2a-Q3 Does the text contain hyponyms or hypernyms of fundamental concept?

2a-Q4 Is there any other information that influenced your decision on criterion 2a?

2b. The text contains at least an implicit endorsement of (2a), that is, the text contains one or more sentences that literally say (modulo synonymy) either that all *Begriffe* that are not *Grundbegriffe*, *Elementarbegriffe* or *höchste begriffe* or *einfache Begriffe*, (*Primum*) *Definitiones/Notiones* (explicit endorsement) or that are *einfacher*, *höher, zusammengesetzt, elementarer*) (implicit endorsement) are composed (*zusammengesetzt, composita*), defined (*definiert, definiendi*) or derived (*abgeleitet, derivantur*) from *Begriffe* that are *Grundbegriffe*, (*Elementarbegriffe, höchste begriffe* or *einfache Begriffe*) or from *einfacher* (*höher, zusammengesetzt, elementarer*). NB: the text might contain hyponyms or hypernyms of *Begriffe*! For instance: *rationata*, *consecutiva*, *definiens* etc.

2b-Questions

2b-Q1 Which words or expressions corresponding to non-fundamental concepts are used?

2b-Q2 Which terms or expressions specify the relation between fundamental concepts and non-fundamental concepts?

2b-Q3 Does the text contain hyponyms or hypernyms of non-fundamental concepts?

2b-Q4 Is there explicit endorsement of a genus/differentia hierarchy of concepts, or model of definitions?

2b-Q5 Which terms of endorsement are used in Q4?

2b-Q6 Is there any other information that influenced your decision on criterion 2b?

We score each condition as follows.

	-1	-0,5	0	+0,5	+1

2a.	explicit rejection	implicit rejection	no info	implicit endorsement	explicit endorsement
2b.	(2a) is at most -0,5 and explicit denial of (2b)	(2a) is at most -0,5 and implicit denial of (2b)	no info	(2a) has at least 0,5 and implicit acceptance of 2b	(2a) has at least 0,5 and explicit acceptance of 2b

1. The full annotation model and scoring system can be found in the document @@, @@ (add description and link). As the reader can tell, our annotation scheme contains philosophical questions and linguistic questions. For the present paper, we will mainly report on the philosophical results of our annotations. In other computational papers, we plan to make use of the more linguistic information we have gathered.
2. On the basis of the annotation model, we annotated, using a Google form, the books on logic and philosophy in our corpus. We iteratively constructed a list of search terms on the basis of which we searched the downloaded pdfs.[14] The results of the annotations are contained in the Spreadsheet Annotation Expert Metadata Spread of the Mathematical Method - Phase II.[15] We recorded the pages of the passages on which our annotations were based, while we also made screenshots of all the pages on which the results of the annotations were based. These screenshots are contained in the folder MM18th-Images.[16] Finally, we saved a copy of all the books we downloaded from Google Books.[17]

4. Quantitative Analysis of the Corpus

4.1 The Numbers

The scores for the annotations of Condition A gave the following results:

TABLE 1

Condition A	-1	-0.5	0	0.5	1
1720s	7,14% (1/14)	14,29% (2/14)	50% (7/14)	14,29% (2/14)	14,29% (2/14)

[14] Links removed for blind review.
[15] Link removed for blind review.
[16] Link removed for blind review.
[17] Link removed for blind review.

1730s	0% (0/23)	0% (0/23)	52,17% (12/23)	17,39% (4/23)	30,43% (7/23)
1740s	0% (0/20)	5% (1/20)	35% (7/20)	15% (3/20)	45% (9/20)
1750s	5,26% (1/19)	5,26% (1/19)	47,37% (9/19)	26,32% (5/19)	15,79% (3/19)
1760s	8,7% (2/23)	8,7% (2/23)	34,78% (8/23)	39,13% (9/23)	8,7% (2/23)
1770s	4,17% (1/24)	4,17% (1/24)	66,67% (16/24)	12,5% (3/24)	12,5% (3/24)
1780s	7,89% (3/38)	5,26% (2/38)	81,57% (31/38)	2,63% (1/38)	2,63% (1/38)

These data show that in the 1720s Wolff's mathematical method was not universally accepted. It was confronted by early criticisms: 14,29% implicitly rejected the mathematical method whereas 7,14% explicitly rejected the mathematical method. From the 1730s to the 1760s Wolff's mathematical method had a relatively high number of proponents, as in these eras the percentage of implicit and explicit acceptance is significantly higher than the percentage of implicit or explicit rejection. In the 1730s, 17,39% implicitly accepted the mathematical method, whereas 30,43% explicitly accepted the mathematical method. By contrast, 0% implicitly or explicitly rejected the mathematical method. In the 1740s, implicit acceptance of the mathematical method was 15%, whereas explicit acceptance stands at 45%. Again, this is higher than the percentages of rejection, with 0% of explicit rejection and 5% implicit rejection. In the 1750s, implicit acceptance of the mathematical method was 26,32%, explicit acceptance was 15,79%, whereas implicit and explicit rejection both scored 5,26%. Finally, in the 1760s implicit acceptance of the mathematical method was 39,13%, explicit acceptance was 8,7%, whereas implicit and explicit rejection both scored 8,7%. In the 1780s, support for Wolff's mathematical method seems to have waned, with the percentage of implicit or explicit rejection being higher than the rates of acceptance. More specifically, explicit rejections was 7,89%, implicit rejection was 5,26%, whereas implicit acceptance was 2,63% and explicit acceptance was 2,63%. Note that the percentage of no info increases with time, which may suggest that the mathematical method lost some of its relevance for late eighteenth-century authors, who increasingly did not discuss it.

The data for condition B are as follows:

TABLE 2

Condition B	-1	-0.5	0	0.5	1
1720s	7,14% (1/14)	14,29% (2/14)	50% (7/14)	28,57% (4/14)	0% (0/14)
1730s	0% (0/23)	0% (0/23)	52,17% (12/23)	17,39 % (4/23)	30,43% (7/23)

1740s	0% (0/20)	5% (1/20)	35% (7/20)	30% (6/20)	30% (6/20)
1750s	0% (0/19)	10,53% (2/19)	52,63% (10/19)	21,05% (4/19)	15,79% (3/19)
1760s	0% (0/23)	17,39% (4/23)	34, 78% (8/23)	34, 78% (8/23)	13,04% (3/23)
1770s	4,17% (1/24)	4,17% (1/24)	66,67% (16/24)	20,83% (5/24)	4,17% (1/24)
1780s	0% (0/38)	13,15% (5/38)	81,57% (31/38)	5,26% (2/38)	0% (0/38)

These numbers support our conclusions drawn from the data of condition A: a relatively high rate of implicit and explicit acceptance of the universality of the mathematical method from the 1730s to the 1760s, followed by a more pronounced rejection of the universality of the mathematical method in the 1780s.

Condition C yielded the following data:

TABLE 3

Condition C	-1	-0.5	0	0.5	1
1720s	14,29% (2/14)	7,14% (1/14)	57,14% (8/14)	7,14% (1/14)	14,29% (2/14)
1730s	0% (0/23)	0% (0/23)	60,87% (14/23)	13,04% (3/23)	26,09% (6/23)
1740s	0% (0/20)	0% (0/20)	40% (8/20)	15% (3/20)	45% (9/20)
1750s	0% (0/19)	5,26% (1/19)	52,63% (10/19)	5,26% (1/19)	36,84% (7/19)
1760s	4,35% (1/23)	8,7% (2/23)	43,48% (10/23)	21,74% (5/23)	21,74% (5/23)
1770s	4,17% (1/24)	0% (0/24)	79,17% (19/24)	8,33% (2/24)	8,33% (2/24)
1780s	0% (0/39)	0% (0/38)	94,73% (36/38)	0% (0/38)	5,26% (2/38)

These data are again consistent with our previous findings.

Condition 2a of the Classical Model yielded the following data:

TABLE 4

Condition 2a	-1	-0.5	0	0.5	1
1720s	0% (0/14)	0% (0/14)	28,57% (4/14)	50 % (7/14)	21,43% (3/14)
1730s	0% (0/23)	0% (0/23)	21,74% (5/23)	47,83% (11/23)	30,43% (7/23)
1740s	0% (0/20)	5% (1/20)	5% (1/20)	45% (9/20)	45% (9/20)
1750s	0% (0/19)	0% (0/19)	10.53% (2/19)	47,37% (9/19)	42,11% (8/19)
1760s	0% (0/23)	0% (0/23)	17,39% (4/23)	60,87% (14/23)	21,74% (5/23)
1770s	0% (0/24)	0% (0/24)	29,17% (7/24)	45,83% (11/24)	25% (6/24)
1780s	0% (0/38)	0% (0/38)	42,1% (16/38)	39,47% (15/38)	18,42% (7/38)

These data show that virtually nobody explicitly or implicitly rejected condition (2a) of the Model. Rather, implicit or explicit acceptance of condition (2a) was consistently high throughout the eighteenth century. The same conclusion can be drawn from the data corresponding to condition 2b of the Model.

TABLE 5

Condition 2b	-1	-0.5	0	0.5	1
1720s	0% (0/14)	0% (0/14)	28,57% (4/14)	50% (7/14)	21,43% (3/14)
1730s	0% (0/23)	0% (0/23)	21,74% (5/23)	47,83% (11/23)	30,43% (7/23)
1740s	0% (0/20)	0% (0/20)	10% (2/20)	50% (10/20)	40% (8/20)
1750s	0% (0/19)	0% (0/19)	10.53% (2/19)	47,37% (9/19)	42,11% (8/19)
1760s	0% (0/23)	0% (0/23)	17,39% (4/23)	60,87% (14/23)	21,74% (5/23)
1770s	0% (0/24)	0% (0/24)	33,33% (8/24)	37,5% (9/24)	29,17% (7/24)
1780s	0% (0/38)	0% (0/38)	52,63% (20/38)	34,21% (13/38)	13,15% (5/38)

The data for condition 3a of the Classical Model looked as follows:

TABLE 6

Condition 3a	-1	-0.5	0	0.5	1
1720s	0% (0/14)	0% (0/14)	28,57% (4/14)	35,71% (5/14)	35,71% (5/14)
1730s	0% (0/23)	0% (0/23)	17,39% (4/23)	21,74% (5/23)	60,87% (14/23)
1740s	0% (0/20)	5% (1/20)	10% (2/20)	15% (3/20)	70% (14/20)
1750s	0% (0/19)	0% (0/19)	5,26% (1/19)	21,05% (4/19)	73,68% (14/19)
1760s	0% (0/23)	0% (0/23)	13,04% (3/23)	43,48% (10/23)	43,48% (10/23)
1770s	0% (0/24)	0% (0/24)	20,83% (5/24)	33,33% (8/24)	45,83% (11/24)
1780s	0% (0/38)	0% (0/38)	23,68% (9/38)	55,26% (21/38)	21,05% (8/38)

Once again, virtually nobody explicitly or implicitly rejected condition (3a) of the Model. Rather, implicit or explicit acceptance of condition (3a) was consistently high throughout the eighteenth century.

The data for condition 3b are as follows:

TABLE 7

Condition 3b	-1	-0.5	0	0.5	1
1720s	0% (0/14)	0% (0/14)	28,57% (4/14)	42,86% (6/14)	28,57% (4/14)
1730s	0% (0/23)	0% (0/23)	17,39% (4/23)	26,09% (6/23)	56,52% (13/23)
1740s	0% (0/20)	0% (0/20)	15% (3/20)	25% (5/20)	60% (12/20)
1750s	0% (0/19)	0% (0/19)	5,26% (1/19)	31,58% (6/19)	63,16% (12/19)
1760s	0% (0/23)	0% (0/23)	13,04% (3/23)	52,17% (12/23)	34,78% (8/23)
1770s	0% (0/24)	0% (0/24)	25% (6/24)	33,33% (8/24)	41,67% (10/24)
1780s	0% (0/38)	0% (0/38)	39,47% (15/38)	42,10% (16/38)	18,42% (7/38)

On the basis of our annotations, we determined which authors were critical of Wolff's mathematical method, i.e., Wolff's specific take on axiomatic science, while endorsing conditions (2) and (3) of the Classical Model. We found 23 authors who fit the bill, and we present them below. This table proves that we should not identify the mathematical method with the traditional axiomatic ideal of science. In the following section, we will provide a qualitative analysis of several of these authors.

TABLE 8

Author	**Year**	**A**	**B**	**C**	**2a**	**2b**	**3a**	**3b**
Rüdiger	1722	-0.5	-0.5	-1	0.5	0.5	0.5	0.5
Hoheisel	1726	-1	-1	-1	0.5	0.5	0.5	0.5
Walch	1726	0.5	0.5	0	0	0	1	0.5
Strähler	1727	-0.5	-0.5	-0.5	0.5	0.5	0.5	0.5
Müller	1733	0.5	0.5	1	0.5	1	1	1
Crusius	1747	-0.5	-0.5	0	0.5	0.5	0.5	0.5
Weis	1747	0.5	0.5	0.5	0.5	0.5	0.5	0.5
Scherffer	1753	-1	-0.5	-0.5	0.5	0.5	0.5	0.5
Wüstemann	1757	-0.5	-0.5	0	1	1	1	1
Dalham	1762	0.5	0.5	0.5	0.5	0.5	0.5	0.5
Scherffer	1763	-1	-0.5	-0.5	0.5	0.5	0.5	0.5
Mangold	1763	0.5	0.5	0.5	0.5	0.5	0.5	0.5
Lambert	1764	-0.5	-0.5	-1	1	1	1	1
Kant	1764	-1	-0.5	-0.5	0.5	0.5	1	0.5
Watts	1764	-0.5	-0.5	0	0.5	0.5	1	1
Unzerin	1767	0.5	0.5	1	0.5	0.5	0.5	0.5
Lambert	1771	-1	-1	-1	1	1	1	0.5
Henning	1774	0.5	0.5	0	0.5	0.5	1	1

Feder	1777	0.5	0.5	0	0.5	0.5	1	1
Steinbart	1780	0.5	0.5	1	1	1	1	1
Kant	1781	-1	-0.5	0	1	1	1	0.5
Heinicke	1788	-0.5	-0.5	0	0.5	0.5	0.5	0.5
Mendelsohn	1786	-1	-0.5	0	0.5	0.5	1	1

5. Qualitative Analysis of Individual Authors

In this section we provide a qualitative investigation of arguments of critics of the mathematical method in eighteenth-century Germany, guided by the results of our quantitative investigation and annotations. We identify two philosophical argumentative patterns in the eighteenth-century that led people to reject the mathematical method. The first is a pattern of arguments developed by people who thought that the nature and role of definitions in mathematics differs from the nature and role of definitions in philosophy. On the basis of this view, many authors argued that the method of mathematics is not identical to the method of philosophy, although they did endorse conditions (2) and (3) of the Model (section 5.1). The second is a pattern of arguments who rejected the universality of the mathematical method because they argued that the mathematical method yields certainty, whereas many philosophical sciences are not completely certain but contain many probable propositions. These authors, as we show, remained committed to conditions (2) and (3) of the Model, and thus assigned all sciences an axiomatic structure, but by rejecting the certainty of the philosophical sciences, they restricted the scope of condition (6) of the Model, according to which all propositions of a proper science are known to be true (section 5.2). Our qualitative investigation is guided by the quantitative investigation and our annotations in the following way: we (i) used the annotations to identify authors who were critical of the mathematical method but nevertheless accepted conditions (2) and (3) of the Model (displayed in Table 8), and (ii) we used the scores of the annotations and annotated passages to identify arguments for rejecting the mathematical method and accepting conditions (2) and (3) of the Model, many of which we describe in detail below. Hence, in what follows we often describe qualitatively the results of our annotations and the annotations guided the subsequent qualitative investigation. We describe the scores of the annotations for individual authors in footnotes. We have also used qualitative research and existing scholarship to inform our research, to structure the results of our annotations, and to obtain more detailed and fine-grained interpretations of the arguments of the authors we discuss, but our annotations were essential to arriving at the research that we report in the following two sections. Hence, what follows is the result of a combined quantitative and qualitative approach.

5.1 The Nature and Role of Definitions in Science

A common argument articulated by critics of Wolff's mathematical method in the eighteenth-century is that the method of providing definitions in mathematics differs from the method of providing definitions in philosophy. This argument is commonly associated with Kant, who articulated it in his *Inquiry Concerning the Distinctness of the Principles of Natural Theology and Morality* (1764) and in the first *Critique* (refs Sutherland, Friedman, de Jong, Gava, others?). In the present section we will show that this argument was common in the eighteenth century, and can be found in the works of Rüdiger and Crusius. Another common argument leveled against Wolff's mathematical method is the claim that the view that axioms and postulates are derived from or grounded by definitions is mistaken. In the following, we demonstrate that this argument was articulated by both Lambert and Kant. Nevertheless, all the critics of the mathematical method we discuss in this section accepted conditions (2) and (3) of the Model.

In *De Sensu Veri et Falsi* (1722), the philosopher Rüdiger rejects Wolff's mathematical method.[18] He states that mathematics has a specific method, which should not be applied to other sciences (Rüdiger 1722, pp. 283, 288). Rüdiger explains that mathematics is the only science that deals exclusively with possible objects, whereas other sciences, e.g. philosophy and physics, deal with real objects (ibid., pp. 285, 286).

This distinction between mathematics and philosophy is also reflected in Rüdiger's accounts of definition. A definition is a concept that explains the nature of what is defined (*definitum*), so that the latter can be distinguished from anything else (ibid., pp. 132, 133). He distinguishes between nominal (*definitio nominalis*) and real definitions (*definitio realis*): the first expresses the meaning of a word, while the second represents the nature of an object, that is its existence or its essence (ibid., pp. 134, 135).

Rüdiger further distinguishes real definitions between logical or metaphysical (*definitio realis logica seu metaphysica*) and causal (*definitio realis causalis*) based on their explanatory role: while the former provides a *reason* (*criterium*) for the existence of the objects, the latter provides the *reason why*, i.e. the actual cause (*causa*), of the existence of the object (ibid., pp. 135, 136). Since logical real definitions do not include the actual cause of the object's existence, it is not possible to prove the real existence of the object from them, but only to postulate it (ibid., p. 137). Philosophy, together with physics and morals, deals with real objects, whose existence needs to be proved (ibid.). On the contrary, mathematics only deals with possible objects, whose existence does not need to be proved but can simply be postulated from the mere possibility of their existence (*ex mera possibilitate sua*) (ibid., p. 285). In this way, Rüdiger explains that, since they deal with different kinds of objects, which involve different kinds of definitions, philosophy and mathematics have a different method.

Nonetheless, Rüdiger accepts conditions (2a) and (2b) of the Model: he takes sciences to have fundamental and non-fundamental concepts (ibid., pp. 132ff, 195ff). He also accepts conditions

[18] The annotations for Rüdiger's *De Sensu Veri et Falsi* (1722) gave the following scores: -0.5 (A), -0.5 (B), -1 (C), +0.5 (2a), +0.5 (2b), +0.5 (3a), +0.5 (3b).

(3a) and (3b) of the Model as he takes that in sciences non-fundamental propositions derive from fundamental ones (ibid., pp. 254, 255, 259).

The philosopher Crusius describes various differences between mathematics and the philosophical sciences, adopting a position that is very similar to that of Rüdiger. [19] In his logic, Crusius writes that philosophy deals with truths that concern natural objects in the world, the essence of natural objects and their causes (Crusius 1747, pp. 3-4). Accordingly, philosophy deals with existing objects and their real grounds (ibid., p. 6). By contrast, pure mathematics, as opposed to applied mathematics, is a science of quantity that does not consider other qualities of real objects (ibid., p. 11). In his *Entwurf*, Crusius states that pure mathematics concerns mere possible objects as opposed to existing objects, such as a point that does not have parts (1766, p. 190). Since mathematics concerns mere possible objects, Crusius warns that it is often illegitimate to use mathematical ideas to infer properties of existing objects (ibid., p. 192).

The differences between mathematics and philosophy come into sharp focus if we consider Crusius' account of definitions. For Crusius, a definition is an abstract concept which suffices to distinguish the object that is referred to from all other objects (1747, p. 843). Crusius follows the traditional Aristotelian account of definitions by arguing that all definitions consist of a genus and specific difference (ibid., p. 845). He further distinguishes between nominal and real definitions. A nominal definition is an abstract concept that signifies the meaning of a word (ibid., p. 855). By contrast, a real definition is an abstract, distinct, and adequate concept of an object (ibid.).

According to Crusius, there are two types of real definitions. There are real definitions of possible objects, which we do not ascribe existence, and which we call ideal definitions (ibid., p. 862). By contrast, there are also real definitions of existing objects, which we call real definitions of real objects (ibid., p. 863). From ideal definitions only hypothetical conclusions follow, which may or may not apply to existing objects (ibid.). In mathematics, which only concerns possible objects, we have ideal definitions, Crusius argues. According to Crusius, we only consider possible objects in mathematics, and we do not need to prove the existence of the definiendum but simply postulate its existence without proving it (ibid., p. 879). In mathematics, it is thus only necessary to prove the possibility of the object defined. The situation is different in the philosophical sciences, in which we have to prove the existence of the definiendum. In philosophy, we strive for real definitions of real objects, and Crusius distinguishes between first concepts or first definitions and inferred definitions, which shows that he accepts conditions (2a) and (2b) of the Model (ibid., p. 865). Crusius stresses that in philosophy we must prove the existence of objects of first concepts (ibid., p. 881). Hence, whereas pure mathematics deals with mere possible objects, and thus only needs to provide ideal definitions of objects, philosophy deals with existing objects, and must strive for real definitions of real objects.

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[19] The annotations for Crusius' *Weg zur Gewißheit* (1747, his Logic) gave the following scores: -0.5 (A), -0.5 (B), 0 (C), 0.5 (2a), 0.5 (2b), 0.5 (3a), 0.5 (3b). The annotations for Crusius' *Entwurf* (1766, his metaphysics) gave the following scores: 0 (A), 0 (B), 0 (c) , 0.5 (2a), 0.5 (2b), 0.5 (3a), 0.5 (3b). On Crusius' position concerning the mathematical method, see also Tonelli 1959, pp. 55-58.

Because the manner in which we provide definitions in pure mathematics and philosophy differs, mathematics and philosophy do not adopt the same method. In his logic, Crusius argues that in mathematics, in contrast to philosophy, we can (i) obtain a definition of an object through one example of an object and (ii) every definition that specifies how an object can be generated can be called a definition (ibid., p. 18). Here, Crusius is calling attention to the special status of constructive definitions in mathematics, as used for example in Wolff's mathematics. In his mathematical writings, Wolff often uses a constructive procedure of generating a mathematical object as a means to define mathematical objects. For example, a circle is defined by letting a straight line CA move around the fixed point C (Wolff [1750] 1999, p. 121). As such, we define a circle on the basis of the construction of *one* object and by specifying the mode of generation of this object. Crusius argues that such constructive definitions are peculiar to mathematics, and that definitions in philosophy are obtained differently. The background of this argument is, as we have seen, that definitions in mathematics differ from definitions in philosophy, the first providing ideal definitions of possible objects and the second providing real definitions of existing objects. For this reason, Crusius could not accept Wolff's claim that the methods of mathematics and philosophy are the same. Nevertheless, as we have seen, Crusius accepted conditions (2a) and (2b) of the Model. He also takes sciences to be based on fundamental propositions from which non-fundamental propositions are derived, thus accepting conditions (3a) and (3b) of the Model (Crusius 1747, pp. 5-6, 64-67, 71-72, 97).

Lambert also rejected the claim that Wolff's method of mathematics is identical to the method of philosophy.[20] According to Wolff's mathematical method, as we have seen, definitions ground axioms and postulates (Wolff 1750 [1999], p. 17). Lambert reverses this order. As has been argued by Heis (2014, pp.616-617), Dunlop (2009, @@), Lawyine (2010, @@), and van den Berg (forthcoming), Lambert claimed that simple concepts cannot be defined. Hence, axioms and postulates, which contain simple concepts, cannot follow from definitions. The function of axioms and postulates is to demonstrate the possibility of complex concepts, which are composed of fundamental concepts. Hence, axioms and postulates precede definitions of complex concepts in terms of simple concepts, and it is not the case, as Wolff argued, that axioms and postulates follow from definitions (Lambert 1771). Nevertheless, Lambert followed the Model (see for extensive discussion of Lambert's axiomatic Wolters 1980). He took sciences to have fundamental concepts, which he called foundational concepts (*Grundbegriffe*) or simple concepts (*einfache Begriffe*). Non-fundamental concepts are called *Lehrbegriffe* (by analogy with theorems (*Lehrsätze*)) or *zusammengesetzte Begriffe* and are composed of or defined in terms of the fundamental concepts (Lambert 1764, p. 420-422). Moreover, Lambert also accepts conditions (3a) and (3b) of the Model insofar as he takes sciences to be based on axioms and postulates from which non-fundamental propositions are derived (Van den Berg (forthcoming))).

Kant's account of definitions in philosophy and mathematics takes up insights of both Crusius and Lambert, although Kant probably arrived at his views independently. In his *Inquiry*

[20] The annotations for Lambert's *Neues Organon* (1764) gave the following scores: -0.5 (A), -0.5 (B), -1 (c) , 1 (2a), 1(2b), 1 (3a), 1 (3b). The annotations of Lambert's *Architektonic* (1771) gave the following scores: -1 (A), -1 (B), -1 (C), 1 (2a), 1 (2b), 1 (3a), 0.5 (3b).

(1764), Kant rejected the idea that Wolff's mathematical method is identical to the method of the philosophical sciences.[21] This claim was based, like Crusius, on the idea that the methods of providing definitions in mathematics and philosophy are different. Kant argued that mathematics obtains all of its definitions synthetically, i.e., through the arbitrary combination of concepts. Thus, to give Kant's own example, we can arbitrarily combine concepts and think of "four straight lines bounding a plane surface so that the opposite sides are not parallel to each other" (AA 2: 276) and call this figure a *trapezium*. By contrast, philosophy does not arrive at its definitions synthetically, but obtains definitions by *analyzing* concepts, thus obtaining characteristic marks and making the analyzed concept complete and determinate (Ibid.). In addition, Kant argues that mathematics, in its analyses, proofs, and inferences, examines the universal under signs in concreto (AA 2: 278). Thus, for example, in order to discover the properties of a circle, we construct or draw one circle, and proceed to prove properties about all circles on the basis of this construction and auxiliary constructions (ibid.). Here, Kant, like Crusius, points to the importance of construction in mathematics, including constructive definitions, and contrasts this practice to philosophy. In philosophy, we do not proceed via construction, i.e., by examining the universal in concreto, but we analyze words (the signs of philosophy) which signify universal concepts, and represent the universal *in abstracto*, without appealing to constructions (the individual and concrete signs of mathematics) (AA 2: 279). On the basis of these and other methodological differences between philosophy and mathematics, Kant rejected the idea that the mathematical method is identical to the method of philosophy.

In his *Inquiry*, Kant also articulates the idea, like Lambert, that axioms or definitions precede and ground definitions of concepts. According to Kant, as we have seen, a fundamental difference between philosophy and mathematics is that mathematics arrives at its definitions synthetically whereas philosophy arrives at its definitions analytically (AA 2:276). Kant concludes from this fact that in philosophy, definitions constitute the end-point of enquiry and cannot constitute the starting-point of investigation, as in mathematics (as has also been stressed by Friedman 1992, @@, and Sutherland 2010). Kant thus rejects Wolff's view that definitions in philosophy ground a science in the sense that axioms and postulates are derived from them. Rather, according to Kant definitions are made possible on the basis of so-called indemonstrable judgments:

> In philosophy, where the concept of the thing to be defined is given to me, that which is initially and immediately perceived in it must serve as an indemonstrable judgment. For since I do not yet possess a complete and distinct concept of the thing, but am only now beginning to look for such a concept, it follows that the fundamental judgement cannot be proved by reference to this concept. On the contrary, such a judgement serves to generate this distinct cognition and produce the definition sought. (AA 2: 281-282)

[21] The annotations for Kant's *Inquiry* (1764) gave the following scores: -1 (A), -0.5 (B), -0.5 (C), 0.5 (2a), 0.5 (2b), 1 (3a), 0.5 (3b).On Kant's Inquiry, see Tonelli 1959, pp. 64-66.

Hence Kant argues, like Lambert, that in philosophy indemonstrable judgments precede and produce definitions (see also AA 2: 285). As an example, Kant cites indemonstrable judgments concerning space, e.g., the judgments that (i) there is a manifold in space of which the parts are external to each other, (ii) that this manifold is not constituted by substances, and (iii) that space has three dimensions (AA 2: 281). Such judgments, Kant seems to argue, demonstrate certain characteristic marks of the concept of space and thus allow us to arrive at a distinct, complete, and precise concept of space, i.e., a definition.

Although Kant rejects Wolff's mathematical method, he does accept the Model.[22] Thus, in the *Inquiry* he claims that philosophy possesses unanalysable concepts (AA 2: 279), and importantly, assigns the synthetic method of composing composite concepts from fundamental concepts to a future metaphysics. As Kant puts the point:

> Metaphysics has a long way to go yet before it can proceed synthetically. It will only be when analysis has helped us towards concepts which are understood distinctly and in detail that it will be possible for synthesis to subsume compound cognitions under the simplest cognitions, as happens in mathematics (AA 2: 290).

Hence, in a future scientific metaphysics, which Kant aims to establish, we can synthetically compose composite concepts from simple concepts. As such, Kant accepted conditions (2a) and (2b) of the Model. Evidence for Kant's acceptance of conditions (3a) and (3b) is given by the fact that he takes metaphysics to be based on indemonstrable fundamental truths (AA 2: 281), and that such truths are the basis for *inferring* or *demonstrating* non-fundamental truths (AA 2: 294-296).

In eighteenth-century Germany, we thus find, despite widespread acceptance of conditions (2) and (3) of the Model, a critique of Wolff's mathematical method because (i) the method of providing definitions in mathematics was taken to differ from the method of providing definitions in philosophy, and (ii) axioms and postulates are, at least in the philosophical sciences, not derived from definitions. These arguments were articulated by multiple authors and are not unique to Kant. In the next section, we will present another widespread argument for being critical of Wolff's mathematical method.

5.2 Science and Probability

Another strand of thought in eighteenth-century Germany was critical of the scope of Wolff's mathematical method because the authors argued that this method suggests that all the propositions of sciences treated in accordance with the mathematical method are certain, i.e., known to be true. This idea is problematic, these authors argued, because many propositions of the philosophical sciences are probable. As such, these authors argued that many sciences do

[22] See for Kant's acceptance of the Classical Model in his Critical period, which we do not discuss here, de Jong (2010), van den Berg (2014) and van den Berg (forthcoming).

not satisfy condition (6) of the Model, according to which all propositions of a proper science are known to be true, and they questioned the scope of the Model (see van den Berg & Demarest 2020, pp. 390-391 for a brief exposition of this point). However, as we shall demonstrate in this paper, our quantitative investigation showed that many of these authors did assign to all sciences an axiomatic structure, accepting conditions (2) and (3) of the Model. Our analysis supports the qualitative analysis of Werner Alexander (1996), who describes several German authors, including Christian Thomasius (1655-1728), Andreas Rüdiger (1673-1731), August Friedrich Müller (1684-1761), Adolph Friedrich Hoffman (1703-1741), and Crusius, many of which claimed that demonstrative knowledge was not the only type of scientific knowledge, but that many sciences contained probable knowledge. The fact that our quantitative analysis supports the findings of Alexander demonstrates the reliability of our conclusions. In the following, we describe some of the German authors that we have identified through our investigation, and also highlight several little known authors writing in Latin [check whether Alexander discusses them]

In the *Philosophisches Lexicon* (1726) of Georg Walch, the mathematical method of Wolff, including the claim that axioms and postulates are derived from definitions, is said to be appropriate for mathematics (Walch 1726, pp. 1736-1738).[23] However, Walch rejects the extension of this method to philosophy. According to Walch, the method is not universally applicable and the imitation of this method in philosophy has done great harm in philosophy (ibid., pp. 1738-1739). The major difference between mathematics and philosophy, according to Walch, is that mathematics provides certain truths whereas philosophical truths are very often probable truths (Ibid., p. 2507). Hence, it is the fact that philosophy contains many probabilities that leads Walch to criticize the identification of the mathematical method and the philosophical method. Accordingly, he rejects the idea that philosophical cognition is always certain, i.e., known to be true, which means that Walch rejects condition (6) of the Model for philosophy, and he questions the scope of the Model as a whole. However, although there is no explicit acceptance of condition (2) of the Model, Walch does distinguish between simple concepts and composite concepts (ibid., p. 1504), and argues that sciences must be *gründlich*, according to which both probable and certain truths are derived through correct inferences from principles (*Principia*) or axioms (Ibid., pp. 1369-1370). Hence, Walch accepts that sciences have fundamental propositions and that non-fundamental propositions, whether certain or probable, are derived from these fundamental principles, thus accepting conditions (3a) and (3b) of the Model (Walch makes the same point in his account of demonstration (*Demonstration*), 1726, pp. 486).

A similar position is adopted by the logician August Friedrich Müller, who argues in his *Einleitung der Philosophischen Wissenschaften* ([1728] 1733) that the mathematical method is appropriate for mathematics, but is not universal, insofar as many truths outside of mathematics, such as those of physics, are probable and not certain (pp. 126-127, 638. See also

[23] The annotations of Walch's Lexicon (1726) gave the following scores: 0.5 (A), 0.5 (B), 0 (c), 0 (2a), 0 (2b), 1 (3a), 0.5 (3b). See for an analysis of Walch's general conception of science, van den Berg & Demarest 2020, p. 390.

van den Berg & Demarest 2020, p. 390).[24] Hence, Müller, like Walch, rejected the universal applicability of the mathematical method and argued that condition (6) of the Model is not applicable to all sciences. However, he implicitly accepts conditions (2a) and (2b) of the Model, insofar as he acknowledges the existence of highest concepts, argues that sciences must begin with definitions, and adopts a genus differentia model of definitions ([1728] 1733, pp. 184, 185, 268, 270, 273). Moreover, Müller argues that sciences have fundamental propositions from which non-fundamental propositions are derived, explicitly arguing that this demonstrative method is not peculiar to mathematics, and thus accepts conditions (3a) and (3b) of the Model (ibid., pp. 125, 221, 224, 408-411). Thus, both Walch and Müller stressed the existence of probabilities in the philosophical sciences, while still retaining the idea that sciences have an axiomatic structure in accordance with conditions (2) and (3) of the Model.

Crusius also adopted a position very similar to Walch and Müller (see van den Berg & Demarest 2020, p. 391). Crusius, who, as we have seen in the previous section, was very critical of Wolff's mathematical method, accepted conditions (2) and (3) of the Model, i.e, in science we have fundamental concepts in terms of which we define non-fundamental concepts, and we have fundamental propositions from which we derive non-fundamental propositions (Crusius 1747, pp. 865, 475, 68-79). However, he argued that the philosophical sciences, such as physics, have many probable propositions (1747, pp. 78-79; 1749, 23-24). The reason is that sciences such as physics are axiomatically structured bodies of knowledge, where some of the axioms are probable. Theorems deductively derived from these axioms are therefore also probable (for a more extensive account, see van den Berg & Demarest 2020, p. 391). Hence, although Crusius accepted conditions (2) and (3) of the Model, he rejected condition (6) of the Model for the philosophical sciences, denying that propositions of some philosophical sciences are strictly known to be true (known with certainty).

A similar line of criticism can be found in Weis's *Liber de emendatione intellectu humani* (1747).[25] Weis argues that mathematics works with nominal definitions and axioms, that are certain and do not need proof (*ea per se nota & indemonstrabilis*), and from which certain truths follow (ibid., pp. 131ff, 135, 463, 522, 536ff). Other sciences instead, e.g. physics, work with real definitions, that are uncertain and need to be proved (*demonstranda*, *demonstrabilis*), and from which only probable truths follow (ibid.). Although Weis claims that the mathematical method can still be useful for sciences as physics (ibid., pp. 462, 463), he shows that some sciences deal with probable truths, thus condition (6) of the Model is not valid for all sciences. However, he accepts conditions (2) and (3) of the Model as he explains that sciences have simple concepts, which are responsible for the composition (*compositio*) of complex concepts (ibid., pp. 189ff, 311, 374-376, 469), first principles and axioms (*principia prima*, *generalissima axiomata*) from which everything else can be demonstrated. (ibid., pp. 502ff, 536, 537, 775).

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[24] The annotations for Müller's *Einleitung* (1733) gave the following scores: 0.5 (A), 0.5 (B), 1 (C), 0.5 (2a), 1 (2b), 1 (3a), 1 (3b).

[25] The annotations for Weis's *Liber de emendatione intellectu humani* (1747) gave the following scores: +0.5 (A), +0.5 (B), +1 (C), +0.5 (2a), +0.5 (2b), +0.5 (3a), +0.5 (3b).

In addition, in *Institutiones logicae* (1753), Scherffer remarks that the mathematical method is of little use to all sciences that involve a logic based on probability (*logica probabilium*), e.g. physics. (Scherffer 1753, p.21ff).[26] Thus, Scherffer is critical of Wolff's mathematical method and restricts condition (6) of the Model to sciences that are based on certain principles. Nevertheless, he accepts conditions (2) and (3) of the Model and argues that in science a composite idea (*idea composita*) is composed of simple ideas (ibid., p. 125ff) and that conclusions follow from certain and evident principles (*certis manifestaque principiis)* (ibid., pp. 76, 155ff, 224ff).

Finally, in *De ratione recte cogitandi, loquendi et intelligendi* (1762), Dalham stresses the same point and claims that while the mathematical method is useful for mathematics and disciplines whose demonstrations rely on certain premises, it should not be extended to all sciences (Dalham 1762, pp. 203ff, 250ff).[27] He explains that sciences like theology, philosophy, medicine and law deal with probable truths (*veritates probabiles*) instead (ibid., p. 110, 111). Thus, also Dalham points out that condition (6) of the Model cannot be applied to all sciences. Nonetheless, he accepts conditions (2) and (3) of the Model as he states that sciences have fundamental, simple ideas and less fundamental, complex ideas (ibid., pp. 28ff, 173, 173ff) and fundamental principles, like axioms and definitions, from which non-fundamental propositions follow (ibid., pp. 202ff, 205, 294ff).

In the eighteenth century, we thus see some criticism of the scope of the Model. The scope of the Model was, according to some, limited, because they took the philosophical sciences to contain many probable propositions. Although these authors did assign to all sciences an axiomatic and deductive structure, accepting conditions (2) and (3) of the Model, they denied that sciences such as philosophy and physics could be completely certain, i.e., they severely restricted the scope of condition (6) of the Model.

6. Conclusion

Wolff's mathematical method has been the subject of multiple qualitative studies. However, the precise impact and popularity of Wolff's mathematical method so far remained unknown, since no quantitative studies on this subject exist. In this paper, we have provided the first quantitative study of the popularity and spread of Wolff's mathematical method in eighteenth century German logic and philosophy. On the basis of our quantitative investigation, we reject the claims of interpreters who maintain that Wolff's mathematical method is identical to a traditional axiomatic ideal of science. Rather, we show, by highlighting authors who reject Wolff's mathematical method but endorse a traditional axiomatic ideal of science, that Wolff's mathematical method is a specific take on how to construct an axiomatic ideal of science. Critics of Wolff rejected Wolff's specific take on axiomatics, but not the axiomatic ideal of

[26] The annotations for Scherffer 's *Institutiones logicae* (1753) gave the following scores: -1 (A), -0.5 (B), -0.5 (C), +0.5 (2a), +0.5 (2b), +0.5 (3a), +0.5 (3b).

[27] The annotations for Dalham 's *De ratione recte cogitandi, loquendi et intelligendi* (1762) gave the following scores: +0.5(A), +0.5 (B), +0.5 (C), +0.5 (2a), +0.5 (2b), +0.5 (3a), +0.5 (3b).

science tout court. Finally, our annotations demonstrated, basically without exception, a widespread acceptance of conditions (2) and (3) of the Classical Model, providing the first quantitative support for de Jong & Betti's (2010) qualitative claim that the Classical Model was a widely influential axiomatic conception of science.

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