Bolzano, Kant, and the Traditional Theory of Concepts
A Computational Investigation

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1. Introduction

Recent research shows that valuable contributions are obtained by applying even simple, well-known computational techniques to texts from the history and philosophy of science (van Wierst et al. 2016). In this paper we substantiate the point by relying on computational text analysis in addressing an open question regarding Bernard Bolzano's work on the general methodology of the sciences. We focus on certain aspects of Bolzano's theory of concepts relevant to his theory of *grounding* (*Abfolge*), a sophisticated notion of (non-causal) *scientific explanation*.

Given that grounding ('proving why' or 'giving reasons why') is, traditionally, scientific explanation (Dear 1987: 138, van den Berg 2014: ch 2-3, Beaney 2018), it is rather odd that Bolzano's ideas on grounding are highly appreciated in logic and metaphysics (Correia 2010, Schnieder 2011, Fine 2012, 2017, Rumberg 2013, Poggiolesi 2016a, 2020), but rarely discussed in the philosophy of science. One exception is Roski 2019, who stresses the relevance of Bolzanian grounding for the concept of unification roughly in the sense of a minimal deductive base *à la* Kim (1994; see also Friedman 1974, Kitcher 1989). As Betti 2010 argues, the oddity of the situation is easy to explain as grounding is a form of conceptual, non-causal explanation, and philosophy of science tends to marginalise non-causal explanations; still, restricting the scope of the meaning of 'explanation' to 'causal explanation' is dangerous as it might hamper our understanding of the past, making us miss important insights from a wealth of historical case studies, thus engendering an undesirable separation of philosophy of science from its history.

Bolzano's stance on scientific explanation is due, as known, to his adherence to a (broadly speaking) axiomatic, millennia-old tradition that originated from Aristotle's *Analytica Posteriora* (de Jong & Betti 2010; de Jong 2001; Betti 2010; Lapointe 2010; Roski 2017) and influential upon Descartes, Newton, Kant, Frege, Carnap and Dedekind to name just a few (Losee 2001 ch. 8; Anstey 2017; de Jong 2010; Macbeth 2016; Klev 2011, 2016). Importantly, axiomatics is not a purely logico-mathematical affair: notable axiomatic approaches exist in physics (see Brading & Ryckman 2008 on Hilbert; Glymour & Eberhardt 2016 on Reichenbach), including 20th-century influential takes such as Patrick Suppes' (starting from McKinsey et al. 1953a, 1953b, 1955), and in biology (see Smocovitis 1992 on Woodger and Haldane). Despite claims that axiomatic efforts cannot in fact properly accommodate experimental science (Muller 2011: 97), recent work in the history of biology shows that no such incompatibility in fact exists (van den Berg & Demarest 2020).

According to the axiomatic tradition, a proper science is *cognitio ex principiis*, knowledge from the principles, and this at two levels: at the level of *truths* (true statements), in a proper science certain fundamental truths are the principles from which all other truths follow; at the level of *concepts*, certain fundamental concepts are the building blocks of which all other concepts are composed or defined (de Jong & Betti 2010: 190). Bolzano, famously, was the first to give a thorough account of the key relation among truths in a proper science as *cognitio ex principiis* in his main work, the *Wissenschaftslehre* (1837), namely in terms of the relation of ground and consequence.[i] Grounding among conceptual truths imposes a hierarchy which is related to a hierarchy among concepts. Since grounds are simpler than their consequences, for Bolzano (Roski & Rumberg 2016, section 3.1.), and since conceptual truths consist exclusively of concepts, it follows that concepts themselves are ordered according to simplicity. Yet simplicity is not the only ordering constraint relevant for grounding: grounds are also required to be more general than their consequences; and, importantly, the two constraints of simplicity and generality are sometimes in conflict in Bolzano's writing on grounding (Roski 2017, section 4.4).

Why exactly are simplicity and generality in conflict? No answer to this question is available yet, for little is still known on how Bolzano's truth and concept hierarchies exactly interact. In this paper we contribute to clarifying the latter point by showing that Bolzano acknowledges two non-interdependent orderings of concepts: the ordering by *composition* (from simple to more complex) and the ordering by *subordination* (from general to more particular).[ii] Only the latter, we maintain, is to be identified with Bolzano's hierarchy of concepts.

That this finding is bound not to be only historically relevant might already be clear from its relevance to a proper understanding of Bolzano's grounding and the relevance of the latter for present-day philosophy mentioned above; but there's also a specific and direct repercussion of our results upon present-day research on Bolzano-inspired, proof-theoretic logics of grounding (e.g. Poggiolesi 2016b, 2018, 2020b, Poggiolesi & Nissim forthcoming). Namely, our results offer an alternative philosophical basis for the grounding order with respect to Poggiolesi's account, which relies on conceptual simplicity (Poggiolesi et al., work ongoing, personal communication). One interesting advantage of this possible alternative is that from the modern point of view conceptual generality might also be a technically easier notion to explicate than conceptual simplicity.

To arrive at our results we used a mixed method: an *ideengeschichtlich* model approach coupled with both computational techniques (text mining) and traditional close reading, that is, fine-grained manual textual analysis including following the cross-referencing indicated by the source text. As for the *ideengeschichtlich* model approach: the model approach (which is independent of the use of computational tools) requires using systematised clusters of stable and variable conditions (models) to fix the precise meaning of the concept(ion)s at scrutiny (see Betti & van den Berg 2014). Models in this sense are especially useful to trace (dis)continuities among different thinkers. In this paper we approach the issue of the ordering of concepts in Bolzano by focusing on the continuities between his ideas and Kant's on the so-called 'traditional theory of concepts'. For, notwithstanding the fact that the literature tends to focus on his differences with Kant, Bolzano was heavily influenced by Kant (see e.g. Blok 2016: §4.4); furthermore, Kant’s conception of science, like Bolzano’s, closely fits the traditional ideal of *cognitio ex principiis* (de Jong 2001, 2010), and Kant's theory of concepts follows the traditional doctrine of *praedicabilia* behind the so-called 'tree of Porphyry'.[iii] The latter is a hierarchical ordering of being, and, derivatively, concepts, such that the lower, more complex concepts are composed of higher, simpler ones. Our reasoning is that it is likely that the conceptual apparatus that Kant's ideas presuppose has the right type of compatibility for us to compare the two thinkers fruitfully; in addition, if the traditional theory of concepts plays in Bolzano the role that it plays in Kant's theory of science, then it is prone to be of substantial influence on Bolzano’s views on grounding - a circumstance that thus far has gone unnoticed.

As to the computational techniques: we use an interactive, web-based information retrieval tool with a close reading visualisation interface at front-end called *BolVis*. BolVis has been co-developed by our team for the specific goal of aiding philosophers in analysing unusually extended text collections (van Wierst et al. 2018). Its main functionality is to perform queries on a collection of digitized texts (the *corpus*) and to return the corpus’ most relevant sentences for that query.[iv] The corpus counts about 11,000 pages (about 35%) from the *Bernard-Bolzano-Gesamtausgabe* (BGA), the modern edition of all of Bolzano’s writings, that have been professionally digitized and corrected to 99% accuracy for the sole purpose of internal research use within our team.

Given our methodological and interdisciplinary angle here, the paper is written in a somewhat unusual way for a philosophy paper: for one thing, the philosophical results we obtain computationally are reported in a way that matches their actual discovery, a style reminiscent of use case descriptions in computer science papers focussing on tool evaluation. We do this to highlight the explicit, logbook-style manner of research appropriate to truly interdisciplinary efforts in teams composed of philosophers and computational experts with equal weight. The material is organised as follows. Section 2 introduces the traditional theory of concepts. Section 3 discusses computationally appropriate formulations of our research questions so that they can be addressed with a tool such as BolVis. Section 4 gives a technical description of BolVis for a philosophy audience. Section 5 describes how we use BolVis to answer our research questions, and how it aids text-based philosophical research in general. The Appendix (online only) describes corpus (**A**), algorithmic steps (**B**) and an additional use case (**C**) related to ours for control, i.e. to exclude that the valuable results we get on our use case result from overfitting, i.e. don't generalise.[v]

2. The traditional theory of concepts

De Jong & Betti 2010's *Classical Model of Science* (CMS) models the ideal of science as 'knowledge from the principles' as a system of seven conditions with fixed and variable parts. Three of these conditions concern concepts (de Jong & Betti 2010: 186); here are the first two:

> A science *S* is a proper science according to the CMS, if:
>
> (2a) There are in *S* a number of so-called *fundamental concepts* (or terms).
> (2b) All other concepts (or terms) occurring in *S* are *composed of* (or are *definable from*) these fundamental concepts (or terms).

Conditions 2a-2b express that in a proper or ideal science according to the CMS there is a distinction between fundamental and non-fundamental concepts, where the latter are (ultimately) defined or composed starting from the former. Conditions 2a-2b can be said to capture the common minimal core of the definitional ordering of concepts in any axiomatic conception of science. *A fortiori*, 2a-2b holds also of the so-called *traditional theory of concepts*; what is distinctive of the latter is a specification of 2b to the effect that (i) the definitional order proceeds by term conjunction (logically speaking), or mereological concept-composition (ontologically speaking, i.e. when one considers concepts as a special kind of objects) of simple(r) concepts, so that complex concepts are composed from simple(r) ones; (ii) the composition/conjunction at issue is of a *genus proximum* and a *differentia specifica* according to the tree of Porphyry. In other words, for *A*, *a*, *b* concepts, in the traditional theory of concepts, definitions have this form (de Jong 2010, section 4):

$A = a \oplus b,$

where the (complex) definiendum *A* is a conjunction or composition of the (simpler) definientia *a* and *b*, where *a* is *genus proximum* (with respect to definiendum *A*) and *b* is the *differentia specifica* (with respect to genus *a*). A famous example of such a *genus-differentia* definition is the traditional definition of the species *human* as a *rational* (=*differentia*) *animal* (=*genus*). So, defining *A* (e.g. human) comes down to indicating that the concept-whole *A* is composed of concept-part *a* (*genus proximum*) and concept-part *b* (*differentia specifica*). Note that in this example the genus (*animal*) is a more fundamental concept than the definiendum (though not absolutely fundamental, as it is in turn capable of analysis). Importantly, such *genus-differentia* definitions obey a hierarchical system of concepts ordered by generality: going up in the hierarchy one finds ever higher and more universal genera, going down one encounters ever lower and more specific species (cfr. de Jong 1995: 623). Previous research on Kant has shown that he followed the traditional theory of concepts - a key factor to understanding Kant's famous analytic-synthetic distinction in the context of his philosophy of science (de Jong 1995). This suggests the following specification for CMS for Kant (specification underlined, henceforth 'CMS_k'):

A science *S* is an ideal science according to the CMS_k, if:

($2a_k$) There are in *S* a number of concepts which are fundamental.

($2b_k$) All non-fundamental concepts in *S* are definable as a conjunction/composition of a *genus proximum* and a *differentia specifica*.[vi]

We leave aside here all details related to conjunction/composition issues,[vii] and concentrate on finding the answer to:

(Q) Are, according to Bolzano, all non-fundamental concepts in an ideal science definable per *genus proximum* and *differentia specifica*?

In the next section, we set up our research aimed at answering this question. As we will see, we offer a more articulated and nuanced answer than that given by de Jong & Betti 2010, which is an unargued 'No'.

3. The traditional theory of concepts in Bolzano: setting up the research with BolVis

When relying on a text-mining tool such as BolVis for text-based historico-philosophical research, we need to take into account two basic methodological circumstances related to the fact that BolVis' query input and output are *term-based* (see also the concluding section of Malatierre, Chartier & Pulizzotto, this volume).

(i) Since (Q) is formulated as a concept-based philosophical question, BolVis cannot be used to answer it; (Q) must first be translated into (multiple) term-based questions (van Wierst et al., 2016), including at least:

(Q*) Are there units of text in our Bolzano corpus containing the terms "*genus proximum*" and/ or "*differentia specifica*" or containing semantically related words?

Note that we take it to be a matter of course that it is always possible to pass from questions such as Q to questions such as Q* whenever Q itself can be answered in a traditional way by reading texts: the process of identifying terms that represent in a text the concepts of interest is a process that all of us as philosophers carry through routinely, but also largely implicitly. In computational text analyses however that process needs to be made as explicit as possible.

(ii) We need to identify which other term-based questions like (Q*) we need to answer, which means identifying as many relevant query terms as we can *beforehand*. The more we identify, the larger the chance that we arrive at all the corpus passages pertinent to (Q). Key to this process is an understanding of the context of our questions: the more precise and more in-depth that understanding, the more promising will be our queries. We know e.g. from traditional research that the tree of Porphyry, which relies with modifications on Aristotelian doctrine, comprises five *praedicabilia* (*genus*, *species*, *differentia*, *proprium* and *accident*), so looking for e.g. *proprium* instead of *genus* might give pertinent results as well.

3.1 Context and background: Kant

For an understanding of the context of our questions, we rely on de Jong (1995)'s reconstruction of Kant's account of concepts.[viii] We will see how this helps us identify (potentially) relevant query terms and create hypotheses about Bolzano's account. Kant related the composition of concepts to a famous distinction he introduced between two types of *a priori* judgements: *analytic* and *synthetic* (de Jong 1995, sections 7, 10). Complex concepts, in Kant's view, are composed of other concepts, the *characteristics*, *parts*, or *partial concepts* of that complex concept (de Jong 1995: 622). The proximate parts of a complex concept, also called by Kant the *constitutiva, rationes*, or *essentialia* of that concept, make up the *logical essence* of that concept (de Jong 1995: 633, 635-7) and are always two: a kind or *genus* concept and a specific difference (*differentia specifica*) concept (de Jong 1995: 624, 633).[ix]

In accordance with the tradition, Kant held that concepts are ordered in a hierarchy as 'higher' and 'lower' concepts based on the relation between *genus* and *species*, where 'higher'/'lower' relate to the concepts' position in a Porphyrian tree: a *genus* is a higher concept with regard to its (remote) *species* and a *species* is a lower concept with regard to its proximate *genus* and (if any) its remote *genera* (de Jong 1995: 626). Kant also expresses the hierarchical ordering of concepts in terms of *intension* and *extension* and the relation of *contained in* and *contained under*. Kant calls the collection of partial concepts (proximate as well as remote) of a given complex concept the concept's *intension* and says that these partial concepts are *contained in* that complex concept (de Jong 1995: 622-3, 626). Kant calls the lower concepts relative to a given concept, i.e. that concept's *species*, that concept's *extension*, and says that they are *contained under* that concept (de Jong 1995: 622-3, 626). *Intension* and *extension* are inversely related in Kant's view: if a certain concept is contained in another one, then the latter is contained under the former (de Jong

1995: 626-7). Hence, Kant accepted what is known as the “canon of reciprocity”: the bigger a concept’s intension, the smaller its extension (de Jong 1995: 622-3, 626).

Answering our question Q will establish whether Bolzano’s account of concepts is similar to Kant’s. So far, interpreters have claimed that it rather isn’t. Siebel (2011: 102) e.g. claims - in arguing that Bolzano’s famous criticism of Kant’s distinction between analytic and synthetic judgments is not to the point - that Bolzano misunderstood Kant because he failed to see that Kant adhered to the traditional theory of concepts.[x] Siebel seems thus to imply that Bolzano himself had a different conception of concepts. By contrast, we focus on the similarities between Bolzano and Kant. As we shall see, this enables us to shed new light on Bolzano’s take on CMS_k. In the next two sections, we first explain BolVis’ technology and then present the results we obtained by using it.

4. Method and tool(s)

To obtain the results we present in the next section, we use the following (iterative) workflow: we identify, starting from an abstract philosophical question, promising query terms for the text corpus, and then close read the (sections containing the) sentences returned by BolVis. The text queries we performed are indicated as **query** and **exact query.** In this section we explain in detail these labels and the technology behind them.

Exact query indicates exact string matching searches: given an input query, e.g. *Definition* as in **Exact Query 1** below, BolVis returns sentences containing exactly the string queried, e.g. (i) '*Definition'* or even (ii) '*Def[ini]tio[n]*, but not (iii) *Definitionen* or *definiren* as the corpus is not stemmed, nor (iv) '*Erklärung'*. The query syntax allowed by the tool however admits rather complex exact queries via regular expressions,[xi] and a result like (iii) can be easily obtained this way. **Query**, by contrast, indicates a search functionality that returns results related, but not necessarily identical, to the initial query, such as (iv). E.g. in **Query 1**, we search for “*genus proximum et differentiam specificam*”, and BolVis returns, among other strings of text related to the initial query, one important two-language sentence containing the string "*genus prox.* und *differentia spec*." The language technology behind BolVis explained below makes the **Query** kind of results possible. In particular, note that we did not have to restrict the search to a particular language.

4.1 The Technology behind BolVis: Ariadne

BolVis is a web interface built on top of a text-mining tool called Ariadne, developed at OCLC Research (Koopman et al. 2015; Koopman, Wang, and Englebienne 2019). BolVis makes the technology from Ariadne easily available to users via their web browser, offering a way to navigate the structure of the corpus and close read the text. Several instances of Ariadne exist, one for each corpus of application,[xii] and three different versions exist based on our Bolzano corpus. For this paper, we have used two of them: the web version with BolVis as interface at front-end, and a desktop version from Autumn 2018 with a more basic interface that also allows exact searches. The inner workings of Ariadne are identical. For reasons of space, we confine a step-by-step description of Ariadne's algorithmic procedure to Appendix **B** online, but note that we consider step-by-step descriptions of this type of key methodological importance for tool transparency.

The language technology used by Ariadne is statistical, and relies on the so-called Distributional Hypothesis (Harris 1954; Sahlgren 2008), which states that words with similar meanings tend to occur in similar contexts. Natural Language Processing develops and applies statistical models based on this idea, so-called Distributional Semantic Models (DSMs). While little conceptual work is done in clarifying what aspect of language DSMs exactly capture (Gladkova & Drozd 2016), DSMs prove helpful for finding units of text, such as paragraphs or sentences, that are related to specific search terms in a large corpus, even if the search terms themselves are not used in the relevant paragraph.[xiii]

In general, DSMs rely on the so-called vector space model: the basic tenet behind it is that it is fruitful to model words or documents mathematically as vectors in high-dimensional spaces, and to see certain properties of vectors as representing (measures of) similarity between words. In previous work we described the basics of the vector space model in connection with another software we developed, SalVe, based on that model (van Wierst et al 2016). In explaining the working of Ariadne in this section we take for granted some of the basics just mentioned.

Ariadne uses *count-based* modelling, that is, creates DSMs that rely on counting how often words co-occur with other words in the same unit of text (a sentence, a paragraph, etc.) or word window of a certain size. Let's consider table 1 for an example. In table 1, a vector is represented as an array of numbers modelling the word '*Gattung*' in our corpus (we also say that the vector is *constructed* or *generated* for the word '*Gattung*' by using a certain method). The '*Gattung*'-vector has as many components as the number *n* of (other) words in the corpus (more precisely word types)*:* *n* will also be the number of *dimensions* of the vector space. Since the corpora in DSMs tend to be very large, the number of dimensions of the vector space also tends to be very large (typically a few hundred thousand dimensions). The Bolzano corpus we use in the research for this paper is quite small compared to typical corpora in distributional semantics, but it still has over nineteen thousand different word-types, which implies that the dimensions of the semantic space of the Bolzano corpus is also above nineteen thousand. Table 1 displays seven of the *n* components of the '*Gattung*'-vector (seven of the over nineteen thousand dimensions of the semantic space), associated with the words '*Artunterschied*', *'zur', 'Art', 'Urtheile', 'Urtheilen', 'Theile', 'Beschaffenheiten'.* The numbers specified in the table indicate that '*Gattung*' and '*Artunterschied*' co-occur only five times, while e.g. '*Gattung*' and '*Art*' co-occur 88 times within a certain word window.

Artunterschied	*zur*	*Art*	*Urtheile*	*Urtheilen*	*Theile*	*Beschaffenheiten*	...
5	26	88	15	9	10	19	...

Table 1: *"A count vector for the word 'Gattung'"*

Under the distributional hypothesis we mentioned at the beginning of the section, count vectors - namely vectors the components of which are co-occurrence counts, may be thought of as representations of word usage insofar as (very) similar words will (ideally) have (very) similar vectors. This is also the reason why each vector component is often thought of representing an aspect of the meaning of a word - in some rough sense of 'meaning' related to language use that remains unclear in NLP and related fields.[xiv] We won't attempt to clarify the issue here: we follow practice and talk of 'semantic similarity' between vectors, but it's important to stress that 'semantic similarity' of two words in this context is simply a mathematical measure of distance between two vectors generated for the two words in question given a certain corpus; a widely used distance measure is e.g. cosine similarity (see Gladkova & Drozd 2016 for a useful discussion of similarity and relatedness in distributional semantics models).

Note *en passant* that what we just saw might help clarify one important reason why the corpora used in distributional semantics modelling are typically very large. Expanding on this issue will allow us to introduce information relevant to the specific model created for our corpus. DSMs rely on a notion of semantic similarity based on actual statistics of word usage: this makes data sparsity problematic because small corpora are bound to be data sparse, as data here are occurrences of words in context. Consider the vector *v* of a word *A* occurring only a few times in a corpus of several million (token) words: *v* will be sparse, as co-occurrence counts in every dimension will be low. Suppose now that *A* is, on an intuitive understanding of meaning, very similar in meaning to another word B, which however occurs far more frequently in the corpus. The vectors for A and B would be quite different from each other, despite the fact that A and B's are close in meaning in an intuitive sense, because the corpus contains too little evidence for all the possible dimensions of the infrequent word. To obviate this problem, in most DSMs co-occurrence counts are normalized to control for word frequency. This also happens in the Ariadne model of our Bolzano corpus.

However, the larger the corpora, the higher the number of dimensions of the vector space. Since systems that need to compute a very high number of dimensions are computationally inefficient, techniques of dimension reduction are usually applied, as a smaller semantic space (i.e. the geometrical entity defined by the amount of dimensions) is easier to process. Dimension reduction consists, very roughly, in removing some of the over nineteen thousand columns in Table 2, while aiming to still preserve distance relations among vectors (and thus among words).

Preserving distance among vectors is important because, as we have seen, vector distance is what represents similarity relations among words as they are used in the corpus: removing dimensions from vectors means influencing distance among them and thus the representation of similarity relations among vectors. Under certain conditions, distance is generally considered as preserved. It remains difficult, though, to have any intuitive understanding of which ‘aspects of meaning’, if any, are captured by vectors that undergo dimension reduction. This is because vectors are then created from highly compressed information, and because the compression in question typically makes use of randomization techniques that influence the end result. Dimension reduction is also applied in the Ariadne model for our corpus.

What is the impact of normalisation or dimension reduction for our case? This is a question we aren't yet in a position to answer satisfactorily. Research in applying various types of DS modelling to philosophical corpora as small as ours is in its infancy, and it generally does not include fine-grained investigations at the detail of tweaking e.g. several different types of dimension reduction used (or not using any), the removal of very infrequent words (or not removing anything). We will come back to this very important point - proper evaluation - in the conclusion.

5. Results

In this section, we show how we use BolVis to answer our question:

(Q) Are, according to Bolzano, all non-fundamental concepts in an ideal science composed of (definable per) a *genus proximum* and a *differentia specifica*?

As we saw in section 3 above, a natural first step to take in order to find the answer - i.e. discover to which extent Bolzano adhered to CMS_k - is to check if Bolzano discussed (Q) explicitly, and if so, what he wrote about it. To this aim, we initially focus on (Q*):

(Q*) Are there units of text in our Bolzano corpus containing the terms “genus proximum” and/ or “differentia specifica” or containing semantically related words?

Query 1. We query BolVis for “*genus proximum et differentia specifica*”. BolVis returns a list with sentences,[xv] of which we skim the first 50. Of these, 1-6 actually contain the terms “*genus proximum*” and “*differentia specifica*”, while 12 contains those terms abbreviated (“*genus prox.* und *differentia spec.*”); the remainder are sentences (partly) in Latin. The sentences containing the query terms (or their abbreviations) all seem relevant, especially the second on the list, this sentence from *Wissenschaftslehre* (WL), §559:

> "Unrichtig, und zum Theile schon mit den beiden vorigen widerlegt, däucht mir auch der fast allgemein angenommene Kanon, daß eine jede Erklärung aus genus proximum und differentia specifica bestehen müsse."

Here, Bolzano writes that to him it seems wrong and already partially refuted by what he wrote before, that, as is commonly accepted, every definition should consist of a *genus proximum* and a *differentia specifica*. This seems to be a good starting point for our research, so we turn to the source text.

Close reading. In WL, section 559 (henceforth WL§559), Bolzano discusses existing accounts of definition (his term is "*Erklärung"*) and from this we learn some important things about Bolzano's own views.[xvi] We learn that in Bolzano's view definitions of a certain concept give the parts of that concept, i.e. its partial concepts ("*Bestandtheile"*), plus the way in which these parts are connected in that concept. A definition (the *definiens*) is not merely equivalent to the concept it defines (the *definiendum*), but is *identical* to it in Bolzano's view, and thus every complex concept has exactly one definition. About *genus-differentia* definitions specifically, Bolzano writes that it is not the case that every definition consists of a *genus proximum* and a *differentia specifica*, because such definitions do not work for certain kinds of concepts: negative, objectless, and imaginary concepts. Later on, in WL§559.13, Bolzano writes that even for concepts that are highly complex, it is not wrong to say that they consist of two parts: a *genus proximum* and a *differentia specifica*.

So far Bolzano seems to tweak (rather than outright reject) condition 2b-2 of CMS_k. It remains to be clarified, for cases in which according to Bolzano definitions may be considered of the *genus-differentia* type, what role he attributes to the *genus* and *differentia* concepts. Specifically: do *genus* and *differentia* concepts determine the conceptual hierarchy captured in the CMS_k, as they do for Kant and the tradition? The third result of the previous query seems interesting: a sentence of WL§509 in which Bolzano writes that one cannot find the highest *genus* ("*höchsten Gattung*") by means of identifying *genus proximum* and *differentia specifica*, but by identifying that simple concept which has the largest extension (*"weitesten Umfange"*).

Query 2. To gather more evidence on Bolzano's concept of *genus*, we query BolVis for "*Gattung"* (*genus*). Of the first seven sentences that BolVis returns, four of them are from WL§117, so we read that section.

Close reading. In WL§117, Bolzano discusses the ancient doctrine of the universals ("*Universalia oder Praedicabilia*"), and discusses the concepts *genus*, *species*, and *differentia* ("*Unterschied*"). We learn that Bolzano regards the notion of *genus* as highly similar to his own notion of *common concept* ("*Gemeinbegriff*"). However, not all common concepts are *genera*, according to Bolzano, for some common concepts have just two objects in their extension and are therefore not *genera* but *species*, to wit, lowest species. Moreover, the concepts of genus, species, and even that of necessary property ("*proprium*") are relative, in the sense that one can call a certain concept in one context a *genus*, and in another *species* or necessary property, Bolzano writes. Notably, Bolzano writes that many, but not all, *species* concepts are composed of a *genus* concept and a *differentia* concept. As an example, he mentions the concepts *real* ("*wirklich*") and *possible* ("*möglich*"), which he takes to relate to one another as *species* and *genus* and thus as the former being *subordinated* ("*untergeordnet*") to the latter, whereas he takes it not to be the case that the concept *real* consists of the concept *possible* plus something else (a *differentia*). For *any* two simple ("*einfache*") concepts of which one is subordinated to the other, Bolzano adds, it necessarily holds that they have no partial concepts in common at all.[xvii] This is remarkable: apparently, according to Bolzano, it can be that two concepts are simple *and* one is subordinated to the other. This implies that there are different manners to order concepts which are both in line with Bolzano's views: by *composition* and by *subordination*. Our new hypothesis is that there are two orderings of concepts: from more to less general and from simple(r) to (more) complex. We now want to know: which one of these two orderings is correct to take to be "*the* conceptual hierarchy" in Bolzano's view?

Query 3. We query for "*Unterordnung*" (*subordination*) and the first result is the title of WL§97, a section which is dedicated to exactly that notion.

Close reading. We read that section and see that Bolzano explains that a concept *A* is subordinated to a concept *B iff* every object which falls under *A* (i.e. every object to which *A* refers) also falls under *B*, but not *vice versa* (and thus the concept *A* is more particular than *B*, *B* more general than *A*). Importantly, Bolzano here talks of "higher" ("*höher*") and "lower" (*"niedriger"*) concepts: if a concept *A* is subordinated to a concept *B*, then Bolzano says that *B* is a *higher* concept than *A*, *A* a *lower* concept than *B*. Now we want to check whether or not Bolzano talks of "higher" and "lower" concepts also with regard to composition, that is, whether Bolzano would also say that a concept *A* is higher than a concept *B* in case *B* is composed of *A* and something else.

Query 4. To this aim, we do several queries related to the conceptual hierarchy (e.g. "*höher niedriger Vorstellung*" and "*unterordnung allgemein*")[xviii] and for all of them check the first 50 sentences that BolVis returns. In none of these sentences does Bolzano use the words "higher" and "lower" with respect to the composition of concepts (except in a passage from WL§97, in which he discusses a view from other logicians which he explicitly rejects), and in many instead he uses these words again in relation to the notion of subordination. We conclude that according to Bolzano the hierarchy of concepts is governed by subordination, not by composition.

We have seen in section 3 that for Kant the *genus* and *differentia* determine the concept's position within the conceptual hierarchy, and the latter is based on the concepts' intension - and, in virtue of the canon of reciprocity, on extension, too. For Bolzano, to the contrary, as we discussed in the present section, the conceptual hierarchy is based on extension only - namely, via the relation of subordination - and this position is available to Bolzano only in virtue of his rejection of the principle of reciprocity.[xix] The key step here is indeed Bolzano's severance of the two orderings of subordination (related to generality) and composition (related to simplicity):

Bolzano's notion of *genus* and *species* regard only the concept's extension in the Bolzanian sense - that is, the objects falling under the concept -, and it is the concept's extension that determines a concept's place in the conceptual hierarchy; the concept's place in the hierarchy is not (also, and equally) determined via the intensional operation of adding *differentiae* to the (intensional) *genus* as part of the content of a concept, as it is in Kant. However, we have also seen that Bolzano argues that although not all, still many definitions are (or may be considered) of the *genus-differentia* type. What, then, is in his view the role of such definitions? In order to understand this, we first need to get a better idea of what exactly definitions *are* in Bolzano's view.

Query 5. We query for "*definition*" ("*Erklärung*") and obtain several sentences in which Bolzano defines this concept: a definition of a concept *A* gives the parts of *A* plus the way in which these parts are connected in *A* (e.g. WL§§23, 555, 559). However, we note that in writing about definitions, Bolzano uses many words which have to do with the engaging in scientific activity from the subjective point of view of human agents rather than with the objective side of science in itself. For example, in WL§559, Bolzano writes that definitions serve to make concepts (more) *distinct* (*"Verdeutlichung*"), and in WL§23, that definitions amount to *pointing out* ("*Angeben*") the components of a complex concept. Now, Bolzano is famous for having been the first to rigorously distinguish between science in an *objective* sense, i.e. as a body of mind-independent truths and concepts, and science in a *subjective* sense, i.e. as an activity that people engage in, as something presented in textbooks and taught to students, etcetera (de Jong 2001). The distinction between the objective and the subjective order of science is rather clearly reflected in Bolzano's use of language in the two different cases (van Wierst et al. 2016). Therefore, at this point our hypothesis is that definitions, for Bolzano, play a role only on the subjective side, and not on the objective side of science. Now, our query with BolVis does not give any result that explicitly confirms our hypothesis, but it does give many more sentences concerning definitions in which Bolzano again uses words that have to do with science as a subjective activity.

Exact query. In the hope to find more relevant passages using exact search, we query '*Definition*' in exact search and here we find our confirmation. In his philosophical diaries (*Philosophische Tagebücher 1811-1817*), Bolzano writes:

> Defi[ni]tion[en] s[in]d k.[eine] eig[en]tl.[ichen] Urth[ei]le, n[ä]hml[i]ch k[ein]e solch[e]n, [die] [einer] Wiss.[enschaft] an sich g[e]hört[en] sond.[ern] hist.[orische] Urth[ei]le, w[e]lch[e] uns sag[en], d[a]ß wir z[u]r B[e]z[ei]ch[nun]g d[ie]s[e]s o[der] j[ene]s B[e]g.[riffes] d[ie]ß o[der] j[ene]s Z[ei]ch[en] w[ä]hl[en] Urth[ei]l[e], w[e]lch[e] w[o]hl als Hülfs[mi]tt[e]l z[u]r V[e]rst[√n]d[i]g[un]g üb[e]r d[ie] Wiss.[enschaft] nothw.[endig] s[in]d, ab[e]r [ni]cht S[ä]tze d[er] Wiss.[enschaft] s[e]lbst ausmach[en]." (Bolzano 1811-1817: 38-9).[xx]

Even though this passage is from an early date and Bolzano's vocabulary is not yet as exact as later on in the WL, also here he clearly distinguishes between the subjective side (signs, comprehension) and the objective side of science ("science in itself"). And Bolzano expresses very explicitly in this passage that definitions belong to the former, and not the latter. Since this passage confirms the hypothesis we formed based on passages from the WL, we feel confident enough to conclude that Bolzano consistently held that definitions have their place merely on the subjective, and not on the objective side of science, that is, they do not have a place within the objective order of truths, but merely help human beings to discover and understand those truths. This implies that also *genus-differentia* definitions in Bolzano's view belong merely to the subjective side of science. And thus, it seems that in Bolzano's view considering complex concepts as consisting of a *genus* and *differentia* may be useful for, for example, making a number of concepts (not all!) distinct, or for the presentation of truths, but the notions of *genus* and *differentia* do not play a role with respect to the (relations between) truths and concepts in themselves.

Let us finally formulate an answer to our question:

(Q) Are, according to Bolzano, all non-fundamental concepts in an ideal science composed of (definable per) a *genus proximum* and a *differentia specifica*?

No, at least not to Kant's extent, but this does not mean that Bolzano's position is an all-encompassing rupture with Kant's, as suggested in the literature. In particular, Bolzano retains the specific role that the conceptual architecture of the *praedicabilia* plays with respect to the analytic/synthetic and a priori/a posteriori dichotomies.

Let's see what we found out. First note that the term "fundamental" in 2a and 2b has become somehow ambiguous when considering Bolzano's views - or better: 2a and 2b do not tell the whole story. In Kant and the tradition, as we know from de Jong (1995), the hierarchy of concepts is such that the fundamental concepts are the simplest *and* most general concepts. That is, Kant does not (need to) distinguish the conceptual order imposed by generality from the conceptual order imposed by simplicity, because for him complexity and generality are inversely related. But for Bolzano, as we have seen, absolutely simple concepts are not necessarily equally general, and thus Bolzano does (need to) distinguish the two orders. In the order imposed by composition, simple concepts are more fundamental than (more) complex concepts, whereas in the order imposed by subordination, general concepts are more fundamental than (more) particular concepts. But with respect to the former, as we have seen, Bolzano writes that even for concepts which are highly complex, it is (in most cases) not wrong to say that they consist of two parts: a *genus proximum* and a *differentia specifica*. However, such *genus* and *differentia* concepts may serve in Bolzano's view in definitions, and may be epistemically useful to make concepts distinct or in the presentation of a science, but do not play a technical role in the objective order of a science.

Non-fundamental concepts are in Bolzano's view *composed of* or *definable from* fundamental concepts, as required by condition (2b) of the CMS_k, only when fundamental is to mean only the simplest, and not (also) the most general concepts. But our research has revealed that Bolzano speaks of higher and lower concepts only when discussing the order imposed by subordination, and not when discussing the one imposed by composition. This suggests that, even though Bolzano certainly accepts conditions (2a) and (2b), these conditions for Bolzano do not capture "*the* conceptual hierarchy", for by this he means the hierarchy of concepts in terms of subordination, i.e. generality. We take that it is this ordering that is relevant for grounding. This has important repercussions for existing accounts of grounding, and any present-day research relying on them, and opens up a substantial line of research which we will pursue in future work.

6. Conclusion and further work

We have argued that Bolzano's view is a modification of the traditional theory of concepts as follows: in a proper science *S*

($2a_B$) There are a number of concepts in *S* which are fundamental in the (mereological) sense of being *simple(st)* (of no/least complexity), and a number that are fundamental in the (quasi-semantic) sense of being *most general* (least particular).

($2a_{Bbis}$) Among simple(st) concepts, some are more general than other concepts.

($2b_B$) Not all, though likely most, of the non-fundamental concepts in *S* in the mereological sense, i.e. the (more) complex ones, are definable per *genus proximum* and a *differentia specifica*.

To show this, we have applied a mixed method including a close reading tool, BolVis, to a corpus of a large part of Bolzano's writings. Searching on BolVis for freely specified fragments of texts returned results ranked by highest semantic similarities with respect to the initial query in the sense we explained in section 4.

BolVis has been both complementary and crucial to our investigation, by enabling in practice the examination of a far larger volume of text than is usual in more traditional modes of research in philosophy. The advantages of this approach for philosophy and related disciplines are undeniable. We have valuably used Ariadne's technology to find out which words or fragments of text are most similar to a certain input (query) - that is, words with vector representations with a high cosine similarity - even if the words that are used in the text are completely different from the query.

Note however that the language technology behind BolVis, Ariadne, has never been properly evaluated against expert judgements from philosophy or related domains. That is, whereas Ariadne has received evaluation against generic technical benchmarks such as STS (semantic textual similarity, cf. Koopman, Wang, and Englebienne 2019),[xxi] methodological questions arising from our specific computational procedure for corpora such as ours do not yet have a proper answer. We don't know e.g. why exactly we got results mostly from the *Wissenschaftslehre*. The point is that we can't take this at face value.[xxii] We don't know whether we missed out on important passages that the system did not output (recall) - or did output, but were misranked from the viewpoint of the philosophers' judgment of relevance (precision). We do know that some of the output passages were, for the philosophers, irrelevant, and that we generally cherry-picked the relevant ones. Whereas fixing standards of relevance for the field which can approximate expert judgement is highly challenging, it is also the only way to increase trustworthiness of computational text analysis: we need to design experiments to make these standards explicit and measurable. Our use of models is a step in this direction, but going beyond the kind of work we do in this paper requires that we provide annotations for model variants such as CMS_K and CMS_B that are more specifically of a linguistic type. This will be a topic for future work.

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References

Anderson, R. Lanier. “It Adds Up After All: Kant’s Philosophy of Arithmetic in Light of the Traditional Logic.” *Philosophy and Phenomenological Research* 69, no. 3 (2004): 501–540.

Anderson, R. Lanier. “The Wolffian Paradigm and Its Discontent: Kant’s Containment Definition of Analyticity in Historical Context.” *Archiv für Geschichte der Philosophie* 87, no. 1 (2005): 22–74.

Anstey, Peter R. *The Idea of Principles in Early Modern Thought: Interdisciplinary Perspectives*. Taylor & Francis, 2017.

Beaney, Michael, "Ancient Conceptions of Analysis" in "Analysis", *The Stanford Encyclopedia of Philosophy* (Summer 2018 Edition), Edward N. Zalta (ed.), URL = <https://plato.stanford.edu/archives/sum2018/entries/analysis/>.

Berg, Hein van den. *Kant on Proper Science: Biology in the Critical Philosophy and the Opus Postumum*. Springer 2014.

Berg, Hein van den, and Boris Demarest. “Axiomatic Natural Philosophy and the Emergence of Biology as a Science.” *Journal of the History of Biology*, June 16, 2020. https://doi.org/10.1007/s10739-020-09609-2.

Betti, Arianna. “Explanation in Metaphysics and Bolzano’s Theory of Ground and consequence.” *Logique et Analyse* 211 (2010): 281–316.

Betti, Arianna, and Hein van den Berg. “Modelling the History of Ideas.” *British Journal for the History of Philosophy* 22, 2014: 812-835.

Blok Johan. *Bolzano's Early Quest for a Priori Synthetic Principles: Mereological Aspects of the Analytic-synthetic Distinction in Kant and the Early Bolzano*. University of Groningen; 2016.

Bolzano, Bernard (1803-1817). *Bernard Bolzano Gesamtausgabe / Reihe II: Nachlass / Wissenschaftliche Tagebücher / Philosophische Tagebücher 1811-1817 / Philosophische Tagebücher 1803-1810. Erster Teil: Abt. B /BD 16*. Edited by Jan Berg. Stuttgart-Bad Cannstatt: frommann-holzboog, 1981.

Brading, Katherine A., and Thomas A. Ryckman. “Hilbert’s ‘Foundations of Physics’: Gravitation and Electromagnetism within the Axiomatic Method.” *Studies in History and Philosophy of Science Part B: Studies in History and Philosophy of Modern Physics* 39 (2008): 102–53.

Centrone, Stefania. “Der Reziprozitätskanon in den Beyträgen und in der Wissenschaftslehre.” *Zeitschrift Für Philosophische Forschung* 64 (2010): 310–30.

Dear, Peter. “Jesuit Mathematical Science and the Reconstitution of Experience in the Early Seventeenth Century.” *Studies in History and Philosophy of Science Part A* 18 (1987): 133–75.

Fine, K. "Guide to ground." In Correia, F. and Schnieder, B., editors, *Metaphysical grounding*, Cambridge University Press, 2012: 37–80.

Friedman, Michael. “Explanation and Scientific Understanding.” *The Journal of Philosophy* 71 (1974): 5-19.

Gladkova, A., & Drozd, A. “Intrinsic evaluations of word embeddings: What can we do gobetter?” In *Proceedings of the 1st Workshop on Evaluating Vector-Space Representations for NLP* (2016): 36–42.

Glymour, Clark, and Frederick Eberhardt. “Hans Reichenbach.” In *The Stanford Encyclopedia of Philosophy* (Ed Zalta ed.), Winter 2016. https://plato.stanford.edu/archives/win2016/entries/reichenbach/.

Harris, Zellig S. “Distributional Structure.” *Word* 10 (1954): 146–162.

Herbelot, Aurélie, Eva von Redecker, and Johanna Müller. “Distributional Techniques for Philosophical Enquiry.” In *Proceedings of the 6th Workshop on Language Technology for Cultural Heritage, Social Sciences, and Humanities*, 45–54. Association for Computational Linguistics, 2012.

Jong, W. R. de. “Kant’s Analytic Judgments and the Traditional Theory of Concepts.” *Journal of the History of Philosophy* 33 (1995): 613–41.

Jong, W. R. de. “The Analytic-Synthetic Distinction and the Classical Model of Science: Kant, Bolzano and Frege.” *Synthese* 2007 (2010): https://doi.org/10.1007/s11229-008-9420-9.

Jong, W. R. de. "Bernard Bolzano, Analyticity and the Aristotelian Model of Science." *Kant-Studien* 92 (2001): 328–49. https://doi.org/10.1515/kant.92.3.328.

Jong, Willem R. de, and Arianna Betti. "The Classical Model of Science: A Millennia-Old Model of Scientific Rationality." *Synthese* 174 (2010): 185–203.

Kitcher, Philip. "Explanatory Unification and the Causal Structure of the World", in *Scientific Explanation*, P. Kitcher and W. Salmon. Minneapolis: University of Minnesota Press, 1989: 410–505.

Klev, Ansten. "Carnap on Unified Science." *Studies in History and Philosophy of Science Part A* 59 (2016): 53–67.

Klev, Ansten. "Dedekind and Hilbert on the foundations of the deductive sciences." *The Review of Symbolic Logic* 4 (2011): 645–81.

Koopman, Rob, Shenghui Wang, and Gwenn Englebienne. "Fast and Discriminative Semantic Embedding." In *Proceedings of the 13th International Conference on Computational Semantics - Long Papers*, 235–246. Gothenburg, Sweden: ASL, 2019.

Koopman, Rob, Shenghui Wang, and Andrea Scharnhorst. "Contextualization of topics: browsing through the universe of bibliographic information." *Scientometrics* 111 (2007): 1119–1139.

Koopman, Rob, Shenghui Wang, Andrea Scharnhorst, and Gwenn Englebienne. "Ariadne's Thread: Interactive Navigation in a World of Networked Information." In *Proceedings of the 33rd Annual ACM Conference Extended Abstracts on Human Factors in Computing Systems*, 1833–1838. ACM, 2015.

Lapointe, Sandra. "Bolzano a Priori Knowledge, and the Classical Model of Science." *Synthese* 174, (2010): 263–81.

Lapointe, Sandra. *Bolzano's Theoretical Philosophy: An Introduction*. Springer, 2011.

Layritz, Paul Eugen. *Erste Anfangsgründe der Vernunft-Lehre*. Züllichau: Dendeler, 1755.

Losee, John. *A historical introduction to the philosophy of science*. OUP, 2001.

Macbeth, Danielle. "Frege and the Aristotelian Model of Science." In *Early Analytic Philosophy - New Perspectives on the Tradition*, Sorin Costreie ed., Springer, 2016: 31–48.

Meier, Georg Friedrich. *Vernunftlehre*. Halle: J.J. Gebauer, 1762.

Muller, F. A. "Reflections on the Revolution at Stanford." *Synthese* 183: 87–114.

Müller, August Friedrich. *Einleitung in die philosophischen Wissenschaften (Erster Theil)*. 2nd ed. Leipzig: Breitkopf, 1733.

Nunez, Tyke. "Definitions of Kant's Categories." *Canadian Journal of Philosophy* 44 (2014): 631–657.

Poggiolesi, Francesca (2020a). "Logics of grounding." Michael Raven. *Routledge Handbook for Metaphysical Grounding*, Routledge, 2020.

Poggiolesi, Francesca (2020b). "Grounding Principles for (Relevant) Implication." *Synthese*, January 8, 2020.

Poggiolesi, Francesca and Nissim Francez. "Toward a generalization of the logic of grounding.", *Theoria*, forthcoming.

Poggiolesi, Francesca (2018). "On Constructing a Logic for the Notion of Complete and Immediate Formal Grounding." *Synthese* 195 (2018): 1231–54.

Poggiolesi, Francesca (2016a). "A Critical Overview of the Most Recent Logics of Grounding." In *Objectivity, Realism, and Proof: FilMat Studies in the Philosophy of Mathematics*, Francesca Boccuni and Andrea Sereni (eds.), Cham, Springer 2016: 291–309.

Poggiolesi, Francesca (2016b). "On Defining the Notion of Complete and Immediate Formal Grounding." *Synthese* 193 (2016): 3147–3167.

Recchia, Gabriel. "The Utility of Count-Based Models for the Digital Humanities," 2016. https://www.dhi.ac.uk/dhc/2016/paper/98.

Roski, Stefan. *Bolzano's Conception of Grounding*. Studies in Theoretical Philosophy. Vittorio Klosterman, 2017.

Roski, Stefan, and Antje Rumberg. "Simplicity and Economy in Bolzano's Theory of Grounding." *Journal of the History of Philosophy* 54 (2016): 469–96.

Rumberg, Antje. "Bolzano's Concept Of Grounding (Abfolge) Against The Background Of Normal Proofs." *The Review of Symbolic Logic* 6 (2013): 424–59.

Sahlgren, Magnus. "The Distributional Hypothesis." *Italian Journal of Disability Studies* 20 (2008): 33–53.

Siebel, Mark. "It Falls Somewhat Short of Logical Precision. Bolzano on Kant's Definition of Analyticity." *Grazer Philosophische Studien* 82 (2011): 91–127.

Smocovitis, V. B. "Unifying Biology: The Evolutionary Synthesis and Evolutionary Biology." *Journal of the History of Biology* 25 (1992): 1–65.

Wang, Shenghui, and Rob Koopman. "Semantic Embedding for Information Retrieval." In *BIR@ ECIR*, 122–132, 2017.

Warren, Edward W, and others. *Porphyry the Phoenician - Isagoge*. Vol. 16. PIMS, 1975.

Wierst, Pauline van, Sanne Vrijenhoek, Stefan Schlobach, and Arianna Betti. "Phil@Scale: Computational Methods within Philosophy," CEUR Workshop Proceedings 1681, 2016.

Wierst, Pauline van, Arianna Betti, Steven Hofstede, Thom Castermans, Michel Westenberg, Yvette Oortwijn, Shenghui Wang, and Rob Koopman. "BolVis: Visualization for Text-Based Research in Philosophy." In *3rd Workshop on Visualization for the Digital Humanities*. Berlin, 2018.

Wolff, Christian Freiherr von. *Philosophia rationalis sive logica, methodo scientifica pertractata et ad usum scientiarum atque vitæ aptata: Præmittitur discursus præliminaris de philosophia in genere*. Frankfurt & Leipzig: Renger, 1732.

Zedler, Johann Heinrich. *Grosses vollständiges Universal-Lexicon aller Wissenschafften und Künste* (Vol 7). Halle und Leipzig: Zedler, 1734.

Appendix to:

Ginammi, Annapaola, Jelke Bloem, Rob Koopman, Shenghui Wang, and Arianna Betti. "**Bolzano, Kant and the Traditional Theory of Concepts - A Computational Investigation**". In *The Dynamics of Science: Computational Frontiers in History and Philosophy of Science*, edited by Andreas de Block and Grant Ramsey. Pittsburgh: Pittsburgh University Press, 2021.

The one below is version 3.0, and belongs to the final author version after R&R submitted 12 Sep, 2020. **DO NOT QUOTE WITHOUT CONSENT.**

This Appendix contains (**A**) the description of the corpus, (**B**) the step-by-step algorithmic procedure of Ariadne (**C**) a control use case for Ginammi et al. 2020.

A. **Corpus description**

Arianna Betti, Annapaola Ginammi & Yvette Oortwijn

The corpus for this paper is composed of the following forty-six volumes of Bolzano's oeuvre in critical edition (BGA I = writings; BGA II = Nachlass; BGA III = letters), i.e. just above thirty-five percent of the hundred-thirty volumes planned for the complete edition (of which nineteen volumes of Miscellanea Mathematica).

(1805-1808) Erbauungsreden 1805-1808. Edited by Edgar Morscher and Kurt F. Strasser. Stuttgart-Bad Cannstatt: frommann-holzboog, 2007 (BGA II A 15 = 1 BGA vol.)

(1810-1816) Mathematische und philosophische Schriften 1810–1816. Edited by Jan Berg. Stuttgart-Bad Cannstatt: frommann-holzboog, 1977 (BGA II A 5 = 1 BGA vol.)

(1811-1817) Philosophische Tagebücher 1811 - 1817 Erster Teil. Edited by Jan Berg. Stuttgart-Bad Cannstatt: frommann-holzboog, 1981. (BGA II B 16/1 = 1 BGA vol.)

(1813) Erbauungsreden für Akademiker, Prague: Caspar Widtmann; 2nd improved and enlarged edition: Sulzbach: J. E. v. Seidel, 1839 (BGA I 2 = 1 BGA vol.)

(1817-1827) Philosophische Tagebücher 1817-1827. Edited by Jan Berg. Stuttgart-Bad Cannstatt: frommann-holzboog, 1980 (BGA II B 17 = 1 BGA vol.)

(1824-1848) Briefe an František Příhonský 1824–1848, ed. by Jan Berg, Stuttgart-Bad Cannstatt: Frommann-Holzboog (BGA III, 3/1–3/3 = 3 BGA vols.)

(1827-1844) Philosophische Tagebücher 1827 - 1844 Zweiter Teil. Edited by Jan Berg. Stuttgart-Bad Cannstatt: frommann-holzboog, 1979 (BGA II B 18/2 = 1 BGA vol)

(1828-1840) Zur Physik I 1828-1840. Stuttgart-Bad Cannstatt: frommann-holzboog, 1995 (BGA II B 19 = 1 BGA vol)

(1832-1848) Vermischte mathematische Schriften 1832-1848 I. Edited by Jan Berg. Stuttgard-Bad Cannstatt: frommann-holzboog, 2001 (BGA II A 11/1 = 1 BGA vol)

(1834) [anonymous] Lehrbuch der Religionswissenschaft, ein Abdruck der Vorlesungshefte eines ehemaligen Religionslehrers an einer katholischen Universität, von einigen seiner Schüler gesammelt und herausgegeben, Sulzbach: J. E. v. Seidel; BGA I, 6–8 (=8 BGA vols.)

(1837)
Wissenschaftslehre. Versuch einer ausführlichen und grösstentheils neuen Darstellung der Logik mit steter Rücksicht auf deren bisherige Bearbeiter, 4 volumes, Sulzbach: J. E. v. Seidel; 2nd improved edition: Leipsic: Felix Meiner, 1929, 1929, 1930, and 1931; reprints: Aalen: Scientia, 1970 and 1981; BGA I, 11–14 (=12 BGA vols.)

(1839-1840) Vermischte Schriften 1839-1840, 2 volumes. Edited by Jaromir Louzil. Stuttgart-Bad Cannstatt: frommann-holzboog, 1989 (BGA I 16/1-16/2 = 2 BGA vol)

(1841-1847) Zur Physik II 1841-1847. Edited by Jan Berg. Stuttgart-Bad Cannstatt: frommann-holzboog, 2003 (BGA II B 20 = 1 BGA vol)

(1842-1843) Mathematisch-Physikalische und Philosophische Schriften 1842-1843. Edited by Gottfried Gabriel, Matthias Gatzemeier, and Friedrich Kambartel. Stuttgart-Bad Cannstatt: frommann-holzboog, 1989 (BGA I 18 = 1 BGA vol)

(1845) [anonymous] Ueber die Perfectibilität des Katholicismus. Streitschriften zweier katholischer Theologen; zugleich ein Beitrag zur Aufhellung einiger wichtigen Begriffe aus Bolzano's Religionswissenschaft, Leipsic: Leopold Voss [Bolzano's contributions: pp. 50–117 and 247–399]; BGA I, 19/1–19/2 (=2 BGA vols.)

(1812-1848) Briefe an Josef Sommer und andere. Edited by Jan Berg. Stuttgart-Bad Cannstatt: frommann-holzboog, 2006 (BGA III 5/1 = 1 BGA vol)

(1833-1841) Einleitung in die Größenlehre und erste Begriffe der allgemeinen Größenlehre. Edited by Jan Berg. Stuttgard-Bad Cannstatt: frommann-holzboog, 1975 (BGA II A 7 = 1 BGA vol)

(1833-1841) Größenlehre II: Reine Zahlenlehre. Edited by Jan Berg. Stuttgart-Bad Cannstatt: frommann-holzboog, 1976 (BGA II A 8 = 1 BGA vol)

(1833-1841) Größenlehre IV,1: Functionenlehre. Edited by Bob van Rootselaar. Stuttgart-Bad Cannstatt: frommann-holzboog, 2000 (BGA II A 10,1 = 1 BGA vol)

(1832-1848) Vermischte philosophische und physikalische Schriften. Edited by Jan Berg and Jaromir Louzil. Stuttgart-Bad Cannstatt: frommann-holzboog, 1977-1978 (BGA II A 12/1-12/3 = 3 BGA vols.)

(1846) Sozialphilosophische Schriften. Edited by Jan Berg and Jaromir Louzil. Stuttgart-Bad Cannstatt: frommann-holzboog, 1975 (BGA II A 14 = 1 BGA vol)

B. **Ariadne Step by Step**

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Let's now go back to our Queries: how does Ariadne arrive at the results? Here is a step by step description.

Preprocessing

Step 0. The corpus is preprocessed. In a close reading tool such as BolVis, this step is crucial, because experts need perspicuous identifiers for the segments returned by queries. In other words, experts need to be able to locate the segments in the context of the oeuvre of an author - i.e. they need to know that e.g. the paragraphs (or sentences or sections) returned by queries is paragraph n by author x from work y from year (or date) z. In turn, this requires getting an accurate segmentation of the text not only at the smallest unit level actually needed for Ariadne's computations (in our case sentences), but also at work level and e.g. section level. Note that paragraphs tend to be the most interesting units to take as documents for philosophical analysis[1], but in our case we ended up segmenting at sentence level since, regrettably, most of the files carried no paragraph markers. Except in the case of digital text already available in XML or TEI, Step 0 can take an extraordinary amount of time, especially if the data is of low quality, i.e. contains a high number of OCR mistakes, or has (too) little structure. This is because work at Step 0 must be typically done manually or semi-manually in close cooperation with domain experts. Huge variations in print layout and editorial setup of scanned and OCR-ed corpora pose additional challenges. For this corpus in particular, the substeps included:

0.1

Create a spreadsheet on which the philosophers jotted down manually the correspondence (a) between the files of the scanned volumes (which had non-mnemonic names), volume pages, Bolzano's works, the hierarchical structure of each work, and certain strings serving as delimiters of work structure; (b) between page numbers and the peritextual elements that could be removed by leaving out whole pages such as editors' introductions, frontispieces, and indexes (Bolzano Whattocut.xlsx[2]).

0.2

Create and run a script (segment.py[3]) to separate textual elements from peritextual elements on the basis of 0.1.

0.3

Identify different types of work on the basis of the spreadsheet at 0.1 (cut.txt[4], cut3.txt[5]) and adapt scripting rules manually to them. This was needed because the original scanned volumes contained several different types of work, such as previously published monographs, collections of reviews, collections of letters, and edited manuscripts; furthermore, some files contained multiple work(name)s, some contained only parts of a work. Each type of work got separate scripting rules.

0.4

Formulating general regular expressions scripting rules for sentence and word segmentation for the whole corpus, guided by the assumption that a sentence is anything between two periods (or any of the characters '?', '!') and a word is anything between blank spaces. Every sentence is given an identifier of the form (originalfilename_sectionnumber_paragraphnumber_sentencenumber, e.g. 9783772807992_sect0_para4_sent19)

0.5

The results of 0.4 were evaluated extensively by philosophers in four rounds of in-depth qualitative evaluation, consisting mainly in detecting and listing various types of false positives for sentence segmentation, such as abbreviations ('u. s. w.')', page numbers followed by 'f.', abbreviations of publications titles in German or other languages, citation conventions used in the text ('Joh. Mich. Sailer's Grundlehren der Religion'), and references to other sections within sections. The feedback also regarded problems created by the presence of various peritextual elements and attributable to layout design, use of various editorial markers and typographical conventions (editorial footnotes, editorial apparatus, author's footnotes, original page delimiters, page numbers, end-of-line splits, footnote reference markers).[6]

0.6

On the basis of 0.5, exceptions to the general rules were formulated, including:

1. Loading 598 abbreviations (abrevs.txt[7]) as exceptions to sentence segmentation;
2. Not segmenting within angular (<>), square ([]) and curly ({}) brackets;
3. Remove footnotes (no full success);
4. Remove references in the text (no full success);

The script doing 0.2, 0.4 and 0.6 is segment.py[8]. It generates a sentence directory (sentences_20171108[9]).

0.7

Another script, transform.py[10], was coded that normalises the sentences output in 0.6, i.e. does the following:

1. For certain book types (edited manuscripts), remove the square brackets in the middle of the words.
2. Remove all punctuation and characters of general typography such as paragraph signs (§), angle brackets (〈 〉), double vertical lines (‖), degree signs (°), ring operators (∘) , em dashes (—), guillemets (‹ › « »), and make everything lower case.

For example, transform.py turns this sentence:

9783772807992_sect0_para4_sent19
Im allg.[ememen] ist v.[on] d[e]m B[e]griffe A, im p[a]rt:[ikulären] v.[on] D[in]g[en] (Begriff[en]) d[ie] [un]t[e]r [2] A steh[e]n, d[ie] Rede.

into:

im allgememen ist von dem begriffe a im partikulären von dingen begriffen die unter 2a stehen die rede

3. Combine single letters - originally corresponding to widely spaced characters to mark emphasis - into words. For example, transform.py turns this sentence:

das Unendliche nur w a c h s e n d e n , nie es e r r e i c h e n d e n
Größe selbst bei.

into:

das Unendliche nur wachsenden, nie es erreichenden Größe selbst bei.

which eventually becomes:

das unendliche nur wachsenden nie es erreichenden größe selbst bei

At this point, we have obtained a sentence-split corpus (by segment.py), normalized, in NFKD unicode, lower-cased and transformed in one single txt file (by transform.py).[11] This is the direct input for Ariadne.

Before we turn to the workings of Ariadne, we want to stress that, to our knowledge, no general tool that could aid in the preprocessing task described in this section is as yet available. Existing toolchains that might have been used focus more on the later, linguistic processing steps, such as lemmatizing, part-of-speech tagging and dependency parsing. An example of this is WebLicht (Hinrichs, Hinrichs, and Zastrow 2010), which contains tools for (modern) German, and is user-friendly as it is a web service with a GUI. However, its standard preprocessing toolchain did not let us perform the kind of data wrangling described above, as exemplified by the fact that only a single file can be uploaded at a time. There are more specific tools for early preprocessing tasks such as sentence splitting (Read et al., 2012) that might have been used instead of custom scripts. However, they are typically trained on modern English and do not perform well on historical texts in other languages. Furthermore, not all aspects of the task, such as the removal or isolation of peritextual elements, can be covered in this way. Let’s now turn to Ariadne.

Ariadne

Step 1. All unigrams and bigrams (i. e. n-gram up to n = 2) from the preprocessed corpus[12] are indexed, that is, a list of each (type of) unigram and bigram (henceforth: term) in the corpus is made, and the number of occurrences of each is counted. The output of this step is not saved in a separate document.

Step 2. The index is filtered as follows.[13] Consider any bigram p, i.e. a sequence of two adjoining words p_1 and p_2, with n = total number of documents (in our case, sentences). Ariadne keeps p if ((Count(p)-20)/n)/(Count(p1)/n*Count(p2)/n) > 1, it removes p otherwise. All terms occurring fewer than five times are subsequently removed from the index. A list of 19,297 words and bigrams is thus obtained. This is the output of Step 2.[14]

Step 3. A count vector matrix is created, following - in theory - this process. First, a 19,297 x 19,297 matrix is created, let's call it A = (n x m), with every row of the matrix being a count vector. Then dimension reduction is executed: A is multiplied by a 19,297 x 256 matrix, call it B = (m x p), artificially created by randomly inputting 19,297 x 256 times one of two values +1 or -1. A new 19,297 x 256 matrix C = (n x p) is created by multiplying A and B, obtained by multiplying the entries of the rows of A and those on the columns of B, and summing the products (in this case m = 19,297 products).[15] To be precise, however, Ariadne never actually constructs the full matrix, for efficiency reasons: the results are calculated 'on the fly' so to speak, by first creating B and (the zero matrix) C, then, taking the corpus' sentences one by one, performing the multiplication steps from A x B for each of the terms actually appearing in the sentences and thus update the corresponding rows of C (see Koopman, Wang, and Englebienne 2019 section 3.1 and Algorithm 1 for details). Output of Step 3 are word and bigram vectors for the corpus (these are not saved in a separate document).

Step 4. Normalize the vectors at Step 3 so that they have all magnitude (length) = 1.[16]

Step 5. Typically, at Step 1 of similar text analysis software, words that are highly frequent get removed by resorting to predefined lists of such words (so-called 'stopword lists'). The intuition behind this is that such words (typically 'the', 'for', 'and', etc.) are deemed to have low informative content; by removing them, it is assumed that the vectors are made more distinct, so they can be compared more informatively. Using stopword lists can however be a problem for a number of reasons, the most immediate one being that there might be no reliable lists for the language or the specific domain at issue (in our case 19th century philosophical texts written in the Austrian variant of New High German).[17] Ariadne dispenses with stopword lists and uses instead a corpus-based method for the removal of words relying on so-called orthogonal projection.[18] The method in question consists of two steps. First, Ariadne constructs the average language vector, that is, the mean vector obtained by averaging the components of each term vector in the corpus. Second, the average language vector is subtracted from all vectors constructed so far (that is, projecting all term vectors to the hyperplane which is orthogonal to the average vector).[19] The modelling intuition behind this is that the more (the vector corresponding to) a certain term t is distant in space from the average vector (i. e. the more it differs from it), the more term t is informative. The average language vector can be seen as representing the lowest degree of informativeness in the semantic space. The basic intuition regarding informativeness just mentioned is also behind computational models of text analysis that resort to a fixed list of stopwords, the difference is that in Ariadne the intuition is operationalised, and made relative to a corpus, namely by subtracting the average language vector from all vectors constructed so far (that is, projecting all term vectors to the hyperplane which is orthogonal to the average vector).[20] Ariadne uses the informativeness thus modelled to weigh the importance of a term during queries. One advantage of this method is that it is language-independent, and in fact, even allows the use of multiple languages in one semantic space (Koopman, Wang, and Englebienne 2019).

Step 6. Term vectors are (again) normalized.[21]

Step 7. Document vectors are generated. In our case, vectors for every sentence are generated, by calculating the weighted sum of the term vectors in the sentence, that is, the sum of each term, weighted - i.e. multiplied by the distance of the term vector to the average vector. For instance:

	c_1	c_2	c_3	….	**Weight** (Step 6)
eine	0.56	0.58	0.57	….	0.0427498
besondere	0.89	0.39	0.22	….	0.656847
Gattung	0.39	0.89	0.22	….	0.693849
v_i	0.88	0.90	0.32	….	….

Here is an example of a sentence (document, v_i) of three words (*eine*, *besondere*, *Gattung*). The values of each of the components of v_i (the document vector in the last row) equal the weighted sum of the values of each component (c_1, c_2, c_3) of the three term (here: word) vectors present in the document (note that here we write down only three components, but the real model has n = 256): this means that o.56 is multiplied by 0.0427498, which is its weight (the cosine distances of eine term vectors to the average vector), 0.89 by 0.656847, etc.[22] We can see that 'eine' (typically a term that would appear in every stopword list for German) receives a very low weight, so it does not contribute much to the components of the sentence vector.

Step 8. Normalise sentence vectors to magnitude = 1.

At this point, both terms and sentences are modelled as vectors in the same space and can be compared for similarity when queried.

Now we can see how Queries work. When any string of text is queried:

i. Ariadne computes the vector representation for the query based on the words of the query (weighted sum, same as Step 7); a new vector is created (query vector) that represents the query by taking the weighted sum of the vectors of the words in the query.

ii. Ariadne computes the similarity between the query and all the sentences in the corpus, i.e. it computes which sentences have the highest cosine similarity to the query vector. Then, Ariadne ranks the results, making it possible to see which sentence vector has the highest (cosine) similarity to the query vector.

C. A second use-case: Bolzano's notion of essence

In this section we offer a control use case on Bolzano's concept of essence for the results of the use case on genus and differentia presented in the body of Ginammi et al. (2020). The aim of this section is to give evidence that it is not by accident that the use of BolVis/Ariadne generates valuable results for historico-philosophical investigations.

C.1 Bolzano's notion of essence

While close-reading Section §559 of Bolzano's *Wissenschaftslehre* (1837) (henceforth WL§559) following Query 1 in the main text of Ginammi et al. 2020, we learned that Bolzano distinguishes between essential ("wesentliche") and derivative ("abgeleitete") properties ("Beschaffenheiten") of an object. We hypothesize that this might be important, because we know from de Jong 1995 that for Kant the notion of essence ("Wesen") played an important role with regard to the question of (the possibility of) necessary predication or a priori knowledge, and with regard to his famous analytic-synthetic distinction. Both Bolzano's conception of necessary predication (necessary truth; Rusnock 2012, Siebel 1997, Textor 2013) and his analytic-synthetic distinction are regarded with perplexity by modern interpreters (Rusnock 2013, Siebel 2011; cf. de Jong 2001); if Bolzano has a notion of essence which plays the role it does in Kant, then this circumstance is prone to clarify Bolzano's views on necessary truth and the analytic-synthetic distinction. We pursue this as a second line of computational research for control, and aim to find the answer to:

(Q2) Does Bolzano have a notion of "essence" similar to Kant's?

(Q2) is related to (Q) from the main text, but rests on different evidence (query terms) and therefore serves for control as far as the qualitative performance of BolVis/Ariadne is concerned. As said, we did not evaluate the tool on the basis of a proper ground truth, because such a ground truth for our corpus does not yet exist.

We will discuss Kant's notion of essence in some more detail and then report the results from our research with BolVis aiming at answering (Q2).

C.2 Context and background: Kant

We have seen in section 3.1 of Ginammi et al. 2020 that - according to de Jong 1995, which we will follow also here - in Kant's view the proximate parts of a complex concept, which he also calls the constitutiva, rationes, or essentialia of that concept, make up the logical essence of that concept (de Jong 1995: 633, 635-7) and are always two: a kind or genus concept and a specific difference (differentia specifica) concept (de Jong 1995: 624, 633). In Kant's view, from the logical essence of a concept other characteristics of its objects follow; these characteristics he calls the object's attributes or propria (de Jong 1995: 633). There are attributes of two kinds: first, attributes which are contained in the concept as its remote parts, i.e. parts of the logical essence (so: parts of the genus or differentia), and which are called analytic attributes or analytic propria, and second, attributes which are not contained in the concept, but do follow from it as a consequence, which are called synthetic attributes or synthetic propria (de Jong 1995: 635-8).[23] Note that analytic and synthetic propria follow from the logical essence of a thing in different ways: an analytic proprium can be proven to follow from the logical essence on the basis of the principle of non-contradiction; a synthetic proprium can be proven to follow from the logical essence by means of some (other) kind of deduction (de Jong 1995: 633, 637-8).

The above underlies Kant's famous distinction between analytic and synthetic judgments: judgments in which the predicate is an essentiale or an analytic proprium of the subject, and thus is contained in the subject concept either as a proximate or as a remote part, Kant calls analytic judgments; judgments in which, instead, the predicate is a synthetic proprium of the subject, and thus is not contained in the subject concept but does follow from it, Kant calls synthetic judgments (de Jong 1995: 637). Furthermore, according to Kant, a thing's propria and essentialia together (Kant calls this also the real or natural essence of that thing; de Jong 1995: 635, n. 50) make up all the necessary characteristics of that thing (de Jong 1995: 635-6). We thus see that in Kant's view analytic and synthetic judgments are two kinds of judgments which are a priori or necessarily true:

Type of knowledge	**Corresponding properties**	**Consists of**	**Consists of**	**Predication results in**
A priori, necessary	Real or natural essence	Logical essence	*Genus**	Analytic judgment
			*Differentia**	
		Propria or attributes	*Analytic propria ***	
			*Synthetic propria ***	Synthetic judgment
A posteriori, contingent	*Accident*			
* The parts of the logical essence are also called by Kant the *constitutiva*, *rationes*, or *essentialia* of that concept				
** Analytic *propria* are contained in the logical essence as parts; synthetic *propria* follow as a consequence from the logical essence.				

Table 1: Our reconstruction of Kant's Theory of Concepts from de Jong 1995

We now want to know: how are things in Bolzano? In other words, to what extent does this table represent Bolzano's views?

C3. Results

A natural first step to take is to query for the term "essence".

Query i. A query for "Wesen" (essence) does not give the desired result: almost all sentences that BolVis returns are about the infinite being ("das unendlichen Wesen"), i.e. God (Bolzano was also a priest and theologian). We try essential property ("wesentliche Beschaffenheit"), which works better, and the first four passages we obtain are from WL§111, a section dedicated to the distinction between essential and extra-essential property concepts ("wesentliche und außerwesentliche Beschaffenheitsvorstellungen").

Close reading. We turn to the source text, WL§111, and learn that essential properties are for Bolzano those properties that an object has in virtue of falling under a certain concept, or in other words, properties that follow from the concept; extra-essential properties are properties that do not follow from the concept and thus belong to the object contingently. Thus, like Kant, Bolzano calls properties which a priori can be attributed to an object, i.e. the object's necessary properties, essential properties. But why does Bolzano contrast essential properties to derivative properties in WL§559? It does not seem to be the case that the "derivative properties" of WL§559 are the same as the "extra-essential" properties of WL§111, first, because Bolzano does not write anything to this effect in WL§111 and §559, and second, because we know that derivability ("Ableitbarkeit") is Bolzano's notion of logical consequence, and thus refers to something a priori and necessary - not contingent. So: what exactly are "derivative" properties according to Bolzano?

Query ii. We query for "abgeleitete Beschaffenheit" (derivative property) and get many sentences from Bolzano's mathematical works in which abgeleitet is used in another sense, works which BolVis enables us to filter out, and then obtain the title of WL§113 "Original and derivative property concepts" ("Ursprüngliche und abgeleitete Beschaffenheitsvorstellungen").

Close reading. We read §113 and learn that original properties, which Bolzano also calls constitutive properties, are those properties of a thing which are a part of the concept under which that thing falls; all the other properties are derivative properties. It thus seems that Bolzano's constitutive properties correspond to Kant's constitutiva plus Kant's analytic propria, that is, all parts of a concept, proximate as well as remote. At this point, we expect that Bolzano's derivative properties correspond to Kant's synthetic propria, that is, the properties that are not contained in a concept but that follow from it in some other way. We need confirmation that Bolzano's derivative properties do indeed follow from the concept under which the object falls, or in other words, that the essential properties comprise not only the original properties (Bolzano confirms this explicitly in WL§113), but also the derivative properties (Bolzano writes nothing to this effect in WL§113).

Query iii. Unfortunately, queries for terms related to "derivative property" do not give relevant results - possibly because the results are dominated by sentences from the mathematical works (only 2 out of the 50 results are from non-mathematical works).

Close reading. We turn again to the section in which Bolzano distinguishes between essential and derivative properties of an object (WL§559) and follow his own cross-references, which bring us via WL§515 to WL§502. Here we find what we had been looking for: Bolzano distinguishes between two senses of essence, a broad and a narrow one. The difference between the two Bolzano formulates on the basis of his distinction between grounding and derivability: the essence in the strict sense, which he calls also fundamental essence ("Grundwesen"), comprises those properties that follow from that concept as a consequence from its ground, whereas the essence in the broad sense ("Wesen") comprises also those properties that can be derived from the concept. For example, Bolzano writes, the property of being a system of three points is (or belongs to) the fundamental essence of a triangle (Grundwesen); the property of having angles which sum up to two right ones is a merely derivative property (and thus belongs to the essence in the broad sense - Wesen -, but not to the fundamental essence - Grundwesen) of triangle, because the property of having angles which sum up to two right ones is grounded in the property of being a system of three points, and thus follows from, but is not part of, the concept of triangle itself.

Bolzano writes here explicitly that derivative properties belong to the essence in the broad sense, and thus we conclude that indeed, Bolzano's derivative properties correspond to Kant's synthetic propria.

We are now in a position to answer:

(Q2) Does Bolzano have a notion of "essence" similar to Kant's?

Yes, Bolzano has a notion of "essence" similar to Kant's. For both Kant and Bolzano, in necessary statements the predicate belongs to the essence of the subject. Furthermore, both Kant and Bolzano distinguish a broad and a narrow sense of "essence"; their respective broad and narrow senses of essence are however slightly different. For Kant, the logical essence of an object comprises only the proximate parts of the object's concept, the constitutiva: the genus proximum and the differentia specifica; all other necessary properties, i.e. the propria belong to the real or natural essence. In turn, the propria divide for Kant into analytic propria, i.e. those properties that are contained in the concept as remote parts, and synthetic propria, i.e. those properties that follow from, but are not contained in the concept. For Bolzano, the fundamental essence comprises all properties that are part of the concept (unlike Kant, Bolzano does not distinguish between the proximate and the remote parts of a concept), whereas the essence in the broad sense comprises also all properties that follow from, but are not contained in the concept.

The main difference between Kant and Bolzano is thus that for Kant, the essence in the strict sense, or logical essence, comprises only the constitutiva (i.e. the proximate parts of the concept), whereas for Bolzano, the essence in the strict sense, or fundamental essence, comprises all properties that are parts of the concept (both its proximate and remote parts), that is, Kant's constitutiva plus analytic propria. Note that this makes sense in the light of the results reported in the main text: constitutiva play for Kant a special role which they do not play for Bolzano. Namely, as we have seen in section 3.1 of the main text, for Kant the constitutiva (i.e. the genus and differentia) determine the concept's position within the conceptual hierarchy (where the conceptual hierarchy for Kant is based on the concepts' intension - and, in virtue of the canon of reciprocity, on extension, too), whereas, as we discussed in section 5 of the main text, this is not the case for Bolzano: for Bolzano (who rejects the canon of reciprocity), a concept's place in the conceptual hierarchy is determined by its extension only.

References

Algee-Hewitt, Mark, Ryan Heuser, and Franco Moretti. 'On Paragraphs. Scale, Themes, and Narrative Form'. Pamphlets of the Stanford Literary Lab. Stanford, October 2015. https://litlab.stanford.edu/LiteraryLabPamphlet10.pdf.

Ginammi, Annapaola, Jelke Bloem, Rob Koopman, Shenghui Wang, and Arianna Betti. "Bolzano, Kant and the Traditional Theory of Concepts - A Computational Investigation" [near-to-final version after R&R due Sep 2020]. In *The Dynamics of Science: Computational Frontiers in History and Philosophy of Science*, edited by Andreas de Block and Grant Ramsey. Pittsburgh: Pittsburgh University Press, 2020.

Hinrichs, Erhard, Marie Hinrichs and Thomas Zastrow. "WebLicht: Web-based LRT services for German". In Proceedings of the ACL 2010 System Demonstrations, 25-29. Association for Computational Linguistics, 2010.

Koopman, Rob, Shenghui Wang, and Gwenn Englebienne. “Fast and Discriminative Semantic Embedding.” In Proceedings of the 13th International Conference on Computational Semantics - Long Papers, 235–246. Gothenburg, Sweden: ASL, 2019. https://www.aclweb.org/anthology/W19-0420

Jong, W. R. de. “Kant’s Analytic Judgments and the Traditional Theory of Concepts.” *Journal of the History of Philosophy* 33 (1995): 613–41.

Lapointe, Sandra. “Bolzano’s Semantics and His Criticism of the Decompositional Conception of Analysis.” In *The Analytic Turn*, edited by Michael Beaney, 219. Routledge, 2007.

Lapointe, Sandra. *Bolzano’s Theoretical Philosophy: An Introduction*. Springer, 2011.

Lewis, Edwin Herbert. *The History of the English Paragraph*. The University of Chicago press, 1894.

Mikolov, Tomas, Ilya Sutskever, Kai Chen, Greg Corrado, and Jeffrey Dean. “Distributed Representations of Words and Phrases and Their Compositionality.” ArXiv:1310.4546 [Cs, Stat], October 16, 2013. http://arxiv.org/abs/1310.4546.

Read, Jonathon, Rebecca Dridan, Stephan Oepen, and Lars Jørgen Solberg. “Sentence boundary detection: A long solved problem?”. In Proceedings of COLING 2012: Posters, 985-994, 2012.

Rusnock, Paul. “Remarks on Bolzano’s conception of necessary truth.” *British Journal for the History of Philosophy* 20(4) (2012): 817–837.

Rusnock, Paul. "Kant and Bolzano on analyticity." Archiv für Geschichte der Philosophie 95, no. 3 (2013): 298-335.

Siebel, Mark. “Variation, derivability and necessity.” *Grazer Philosophische Studien* 53 (1997): 117–137.

Textor, Mark. “Bolzano on the source of necessity: A reply to Rusnock.”*British Journal for the History of Philosophy* 21(2) (2013): 381–392.