Appendix to:

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The one below is version 3.0, and belongs to the final author version after R&R submitted 12 Sep, 2020. **DO NOT QUOTE WITHOUT CONSENT.**

This Appendix contains (**A**) the description of the corpus, (**B**) the step-by-step algorithmic procedure of Ariadne (**C**) a control use case for Ginammi et al. 2020.

A. Corpus description
Arianna Betti, Annapaola Ginammi & Yvette Oortwijn

The corpus for this paper is composed of the following forty-six volumes of Bolzano's oeuvre in critical edition (BGA I = writings; BGA II = Nachlass; BGA III = letters), i.e. just above thirty-five percent of the hundred-thirty volumes planned for the complete edition (of which nineteen volumes of Miscellanea Mathematica).

> (1805-1808) Erbauungsreden 1805-1808. Edited by Edgar Morscher and Kurt F. Strasser. Stuttgart-Bad Cannstatt: frommann-holzboog, 2007 (BGA II A 15 = 1 BGA vol.)
>
> (1810-1816) Mathematische und philosophische Schriften 1810 1816. Edited by Jan Berg. Stuttgart-Bad Cannstatt: frommann-holzboog, 1977 (BGA II A 5 = 1 BGA vol.)
>
> (1811-1817) Philosophische Tagebücher 1811 - 1817 Erster Teil. Edited by Jan Berg. Stuttgart-Bad Cannstatt: frommann-holzboog, 1981. (BGA II B 16/1 = 1 BGA vol.)
>
> (1813) Erbauungsreden für Akademiker, Prague: Caspar Widtmann; 2nd improved and enlarged edition: Sulzbach: J. E. v. Seidel, 1839 (BGA I 2 = 1 BGA vol.)

(1817-1827) Philosophische Tagebücher 1817-1827. Edited by Jan Berg. Stuttgart-Bad Cannstatt: frommann-holzboog, 1980 (BGA II B 17 = 1 BGA vol.)

(1824-1848) Briefe an František Příhonský 1824 1848, ed. by Jan Berg, Stuttgart-Bad Cannstatt: Frommann-Holzboog (BGA III, 3/1 3/3 = 3 BGA vols.)

(1827-1844) Philosophische Tagebücher 1827 - 1844 Zweiter Teil. Edited by Jan Berg. Stuttgart-Bad Cannstatt: frommann-holzboog, 1979 (BGA II B 18/2 = 1 BGA vol)

(1828-1840) Zur Physik I 1828-1840. Stuttgart-Bad Cannstatt: frommann-holzboog, 1995 (BGA II B 19 = 1 BGA vol)

(1832-1848) Vermischte mathematische Schriften 1832-1848 I. Edited by Jan Berg. Stuttgard-Bad Cannstatt: frommann-holzboog, 2001 (BGA II A 11/1 = 1 BGA vol)

(1834) [anonymous] Lehrbuch der Religionswissenschaft, ein Abdruck der Vorlesungshefte eines ehemaligen Religionslehrers an einer katholischen Universität, von einigen seiner Schüler gesammelt und herausgegeben, Sulzbach: J. E. v. Seidel; BGA I, 6 8 (=8 BGA vols.)

(1837) Wissenschaftslehre. Versuch einer ausführlichen und grösstentheils neuen Darstellung der Logik mit steter Rücksicht auf deren bisherige Bearbeiter, 4 volumes, Sulzbach: J. E. v. Seidel; 2nd improved edition: Leipsic: Felix Meiner, 1929, 1929, 1930, and 1931; reprints: Aalen: Scientia, 1970 and 1981; BGA I, 11 14 (=12 BGA vols.)

(1839-1840) Vermischte Schriften 1839-1840, 2 volumes. Edited by Jaromir Louzil. Stuttgart-Bad Cannstatt: frommann-holzboog, 1989 (BGA I 16/1-16/2 = 2 BGA vol)

(1841-1847) Zur Physik II 1841-1847. Edited by Jan Berg. Stuttgart-Bad Cannstatt: frommann-holzboog, 2003 (BGA II B 20 = 1 BGA vol)

(1842-1843) Mathematisch-Physikalische und Philosophische Schriften 1842-1843. Edited by Gottfried Gabriel, Matthias Gatzemeier, and Friedrich Kambartel. Stuttgart-Bad Cannstatt: frommann-holzboog, 1989 (BGA I 18 = 1 BGA vol)

(1845) [anonymous] Ueber die Perfectibilität des Katholicismus. Streitschriften zweier katholischer Theologen; zugleich ein Beitrag zur Aufhellung einiger wichtigen Begriffe aus Bolzano's Religionswissenschaft, Leipsic: Leopold Voss [Bolzano's contributions: pp. 50 117 and 247 399]; BGA I, 19/1 19/2 (=2 BGA vols.)

(1812-1848) Briefe an Josef Sommer und andere. Edited by Jan Berg. Stuttgart-Bad Cannstatt: frommann-holzboog, 2006 (BGA III 5/1 = 1 BGA vol)

(1833-1841) Einleitung in die Größenlehre und erste Begriffe der allgemeinen Größenlehre. Edited by Jan Berg. Stuttgard-Bad Cannstatt: frommann-holzboog, 1975 (BGA II A 7 = 1 BGA vol)

(1833-1841) Größenlehre II: Reine Zahlenlehre. Edited by Jan Berg. Stuttgart-Bad Cannstatt: frommann-holzboog, 1976 (BGA II A 8 = 1 BGA vol)

(1833-1841) Größenlehre IV,1: Functionenlehre. Edited by Bob van Rootselaar. Stuttgart-Bad Cannstatt: frommann-holzboog, 2000 (BGA II A 10,1 = 1 BGA vol)

(1832-1848) Vermischte philosophische und physikalische Schriften. Edited by Jan Berg and Jaromir Louzil. Stuttgart-Bad Cannstatt: frommann-holzboog, 1977-1978 (BGA II A 12/1-12/3 = 3 BGA vols.)

(1846) Sozialphilosophische Schriften. Edited by Jan Berg and Jaromir Louzil. Stuttgart-Bad Cannstatt: frommann-holzboog, 1975 (BGA II A 14 = 1 BGA vol)

B. Ariadne Step by Step

Arianna Betti, Jelke Bloem, Rob Koopman & Shenghui Wang

Let's now go back to our Queries: how does Ariadne arrive at the results? Here is a step by step description.

Preprocessing

Step 0. The corpus is preprocessed. In a close reading tool such as BolVis, this step is crucial, because experts need perspicuous identifiers for the segments returned by queries. In other words, experts need to be able to locate the segments in the context of the oeuvre of an author - i.e. they need to know that e.g. the paragraphs (or sentences or sections) returned by queries is paragraph n by author x from work y from year (or date) z. In turn, this requires getting an accurate segmentation of the text not only at the smallest unit level actually needed for Ariadne's computations (in our case sentences), but also at work level and e.g. section level. Note that paragraphs tend to be the most interesting units to take as documents for philosophical analysis[1], but in our

[1] On the paragraph as a unity of thought, see especially Lewis 1894, Ch. 2. We owe the reference to Algee-Hewitt, Heuser, and Moretti 2015. Thanks to Michael Widmer

case we ended up segmenting at sentence level since, regrettably, most of the files carried no paragraph markers. Except in the case of digital text already available in XML or TEI, Step 0 can take an extraordinary amount of time, especially if the data is of low quality, i.e. contains a high number of OCR mistakes, or has (too) little structure. This is because work at Step 0 must be typically done manually or semi-manually in close cooperation with domain experts. Huge variations in print layout and editorial setup of scanned and OCR-ed corpora pose additional challenges. For this corpus in particular, the substeps included:

0.1 Create a spreadsheet on which the philosophers jotted down manually the correspondence (a) between the files of the scanned volumes (which had non-mnemonic names), volume pages, Bolzano's works, the hierarchical structure of each work, and certain strings serving as delimiters of work structure; (b) between page numbers and the peritextual elements that could be removed by leaving out whole pages such as editors' introductions, frontispieces, and indexes (Bolzano Whattocut.xlsx[2]).

0.2 Create and run a script (segment.py[3]) to separate textual elements from peritextual elements on the basis of 0.1.

0.3 Identify different types of work on the basis of the spreadsheet at 0.1 (cut.txt[4], cut3.txt[5]) and adapt scripting rules manually to them. This was needed because the original scanned volumes contained several different types of work, such as previously published monographs, collections of reviews, collections of letters, and edited manuscripts; furthermore, some files contained multiple work(name)s, some contained only parts of a work. Each type of work got separate scripting rules.

0.4 Formulating general regular expressions scripting rules for sentence and word segmentation for the whole corpus, guided by the assumption that a sentence is anything between two periods (or any of the characters '?', '!') and a word is anything between blank spaces.

(@mwidner) for the reference to the latter https://twitter.com/mwidner/status/1148437939527045120?s=20

[2] https://github.com/ariannabetti/eIdeas/blob/master/Bolzano%20Whattocut.xlsx

[3] https://github.com/ariannabetti/eIdeas/blob/master/segment.py

[4] https://github.com/ariannabetti/eIdeas/blob/master/cut.txt

[5] https://github.com/ariannabetti/eIdeas/blob/master/cut3.txt

Every sentence is given an identifier of the form (originalfilename_sectionnumber_paragraphnumber_sentencenumber, e.g. 9783772807992_secto_para4_sent19)

0.5 The results of 0.4 were evaluated extensively by philosophers in four rounds of in-depth qualitative evaluation, consisting mainly in detecting and listing various types of false positives for sentence segmentation, such as abbreviations ('u. s. w.')', page numbers followed by 'f.', abbreviations of publications titles in German or other languages, citation conventions used in the text ('Joh. Mich. Sailer's Grundlehren der Religion'), and references to other sections within sections. The feedback also regarded problems created by the presence of various peritextual elements and attributable to layout design, use of various editorial markers and typographical conventions (editorial footnotes, editorial apparatus, author's footnotes, original page delimiters, page numbers, end-of-line splits, footnote reference markers).[6]

0.6 On the basis of 0.5, exceptions to the general rules were formulated, including:

1. Loading 598 abbreviations (abrevs.txt[7]) as exceptions to sentence segmentation;
2. Not segmenting within angular (<>), square ([]) and curly ({}) brackets;
3. Remove footnotes (no full success);
4. Remove references in the text (no full success);

The script doing 0.2, 0.4 and 0.6 is segment.py[8]. It generates a sentence directory (sentences_20171108[9]).

0.7 Another script, transform.py[10], was coded that normalises the sentences output in 0.6, i.e. does the following:

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[6] https://github.com/ariannabetti/eIdeas/blob/master/First%20sentence%20tokenization%20of%20Bolzano%20corpus.docx

[7] https://github.com/ariannabetti/eIdeas/blob/master/abrevs.txt

[8] https://github.com/ariannabetti/eIdeas/blob/master/segment.py

[9] https://github.com/ariannabetti/eIdeas/blob/master/sentences_20171118.zip

[10] https://github.com/ariannabetti/eIdeas/blob/master/transform.py

1. For certain book types (edited manuscripts), remove the square brackets in the middle of the words.
2. Remove all punctuation and characters of general typography such as paragraph signs (§), angle brackets (⟨ ⟩), double vertical lines (‖), degree signs (°), ring operators (∘), em dashes ( ), guillemets (‹ › « »), and make everything lower case.

For example, transform.py turns this sentence:

> 9783772807992_sect0_para4_sent19
> Im allg.[ememen] ist v.[on] d[e]m B[e]griffe A, im p[a]rt:[ikulären] v.[on] D[in]g[en] (Begriff[en]) d[ie] [un]t[e]r [2] A steh[e]n, d[ie] Rede.

into:

> im allgememen ist von dem begriffe a im partikulären von dingen begriffen die unter 2a stehen die rede

3. Combine single letters - originally corresponding to widely spaced characters to mark emphasis - into words. For example, transform.py turns this sentence:

> das Unendliche nur w a c h s e n d e n , nie es e r r e i c h e n d e n Größe selbst bei.

into:

> das Unendliche nur wachsenden, nie es erreichenden Größe selbst bei.

which eventually becomes:

> das unendliche nur wachsenden nie es erreichenden größe selbst bei

At this point, we have obtained a sentence-split corpus (by segment.py), normalized, in NFKD unicode, lower-cased and transformed in one single txt file (by transform.py).[11] This is the direct input for Ariadne.

Before we turn to the workings of Ariadne, we want to stress that, to our knowledge, no general tool that could aid in the preprocessing task described in this section is as yet available. Existing toolchains that might have been used focus more on the later, linguistic processing steps, such as lemmatizing, part-of-speech tagging and dependency parsing. An example of this is WebLicht (Hinrichs, Hinrichs, and Zastrow 2010), which contains tools for (modern) German, and is user-friendly as it is a web service with a GUI. However, its standard preprocessing toolchain did not let us perform the kind of data wrangling described above, as exemplified by the fact that only a single file can be uploaded at a time. There are more specific tools for early preprocessing tasks such as sentence splitting (Read et al., 2012) that might have been used instead of custom scripts. However, they are typically trained on modern English and do not perform well on historical texts in other languages. Furthermore, not all aspects of the task, such as the removal or isolation of peritextual elements, can be covered in this way. Let's now turn to Ariadne.

Ariadne

Step 1. All unigrams and bigrams (i. e. n-gram up to n = 2) from the preprocessed corpus[12] are indexed, that is, a list of each (type of) unigram and bigram (henceforth: term) in the corpus is made, and the number of occurrences of each is counted. The output of this step is not saved in a separate document.

Step 2. The index is filtered as follows.[13] Consider any bigram p, i.e. a sequence of two adjoining words p_1 and p_2, with n = total number of documents (in our

[11] https://github.com/ariannabetti/eIdeas/blob/master/sentences_20171118.zip (all sentences separately) and https://github.com/ariannabetti/eIdeas/blob/master/input (concatenation of sentences)

[12] https://github.com/ariannabetti/eIdeas/blob/master/input

[13] The formula is adapted from Mikolov et al. 2013 (cf. TrainModel module in code https://github.com/tmikolov/word2vec/blob/master/word2phrase.c, lines 280-25). See also https://en.wikipedia.org/wiki/Pointwise_mutual_information. The addition of -20 has the aim of addressing the fact that PMI *fails* (i.e. becomes unreliable) in case of low frequency (as in Mikolov et al. 2013) as well as the requirement '>1' (instead of 100 as in Mikolov et al. 2013) are set empirically on the basis of observation of the results for a certain corpus, and have so far not be specifically researched for the Bolzano corpus. Note that the Mikolov et al. 2013 formula, though widely used, regards the probability of two words in a document instead of two words in a bigram as one would need here.

case, sentences). Ariadne keeps p if ((Count(p)-20)/n)/(Count(p1)/n*Count(p2)/n) > 1, it removes p otherwise. All terms occurring fewer than five times are subsequently removed from the index. A list of 19,297 words and bigrams is thus obtained. This is the output of Step 2.[14]

Step 3. A count vector matrix is created, following - in theory - this process. First, a 19,297 x 19,297 matrix is created, let's call it A = (n x m), with every row of the matrix being a count vector. Then dimension reduction is executed: A is multiplied by a 19,297 x 256 matrix, call it B = (m x p), artificially created by randomly inputting 19,297 x 256 times one of two values +1 or -1. A new 19,297 x 256 matrix C = (n x p) is created by multiplying A and B, obtained by multiplying the entries of the rows of A and those on the columns of B, and summing the products (in this case m = 19,297 products).[15] To be precise, however, Ariadne never actually constructs the full matrix, for efficiency reasons: the results are calculated 'on the fly' so to speak, by first creating B and (the zero matrix) C, then, taking the corpus' sentences one by one, performing the multiplication steps from A x B for each of the terms actually appearing in the sentences and thus update the corresponding rows of C (see Koopman, Wang, and Englebienne 2019 section 3.1 and Algorithm 1 for details). Output of Step 3 are word and bigram vectors for the corpus (these are not saved in a separate document).

Step 4. Normalize the vectors at Step 3 so that they have all magnitude (length) = 1.[16]

Step 5. Typically, at Step 1 of similar text analysis software, words that are highly frequent get removed by resorting to predefined lists of such words (so-called 'stopword lists'). The intuition behind this is that such words (typically 'the', 'for', 'and', etc.) are deemed to have low informative content; by removing them, it is assumed that the vectors are made more distinct, so they can be compared more informatively. Using stopword lists can however be a problem for a number of reasons, the most immediate one being that there might be no reliable lists for the language or the specific domain at issue (in our case 19th century philosophical texts

[14] https://github.com/ariannabetti/eIdeas/blob/master/vocab
[15] Cf. also https://en.wikipedia.org/wiki/Matrix_multiplication
[16] http://mathworld.wolfram.com/NormalizedVector.html

written in the Austrian variant of New High German).[17] Ariadne dispenses with stopword lists and uses instead a corpus-based method for the removal of words relying on so-called orthogonal projection.[18] The method in question consists of two steps. First, Ariadne constructs the average language vector, that is, the mean vector obtained by averaging the components of each term vector in the corpus. Second, the average language vector is subtracted from all vectors constructed so far (that is, projecting all term vectors to the hyperplane which is orthogonal to the average vector).[19] The modelling intuition behind this is that the more (the vector corresponding to) a certain term t is distant in space from the average vector (i. e. the more it differs from it), the more term t is informative. The average language vector can be seen as representing the lowest degree of informativeness in the semantic space. The basic intuition regarding informativeness just mentioned is also behind computational models of text analysis that resort to a fixed list of stopwords, the difference is that in Ariadne the intuition is operationalised, and made relative to a corpus, namely by subtracting the average language vector from all vectors constructed so far (that is, projecting all term vectors to the hyperplane which is orthogonal to the average vector).[20] Ariadne uses the informativeness thus modelled to weigh the importance of a term during queries. One advantage of this method is that it is language-independent, and in fact, even allows the use of multiple languages in one semantic space (Koopman, Wang, and Englebienne 2019).

[17] For example, Koopman, Wang, and Englebienne 2019 mention some terms that Ariadne identifies as uninformative in their corpus: treatment, reduce, and subsequent. These are not stopwords, but as the corpus used in the paper in question contains exclusively scientific articles from the medical domain, these words occur in almost every text and are therefore uninformative. Downweighing such generic terms makes it easier for the system to find informative and distinctive terms, also in the specific domain of philosophical texts.

[18] https://en.wikipedia.org/wiki/Vector_projection

[19] To understand the notions of vector subtraction and orthogonal projection, the notion of vector decomposition into perpendicular/orthogonal components is useful, see http://www.softschools.com/math/pre_calculus/decomposing_a_vector_into_components/

[20] To understand the notions of vector subtraction and orthogonal projection, the notion of vector decomposition into perpendicular/orthogonal components is useful, see http://www.softschools.com/math/pre_calculus/decomposing_a_vector_into_components/

Step 6. Term vectors are (again) normalized.[21]

Step 7. Document vectors are generated. In our case, vectors for every sentence are generated, by calculating the weighted sum of the term vectors in the sentence, that is, the sum of each term, weighted - i.e. multiplied by the distance of the term vector to the average vector. For instance:

	c_1	c_2	c_3	….	**Weight** (Step 6)
eine	0.56	0.58	0.57	….	0.0427498
besondere	0.89	0.39	0.22	….	0.656847
Gattung	0.39	0.89	0.22	….	0.693849
v_i	0.88	0.90	0.32	….	….

Here is an example of a sentence (document, v_i) of three words (*eine*, *besondere*, *Gattung*). The values of each of the components of v_i (the document vector in the last row) equal the weighted sum of the values of each component (c_1, c_2, c_3) of the three term (here: word) vectors present in the document (note that here we write down only three components, but the real model has n = 256): this means that 0.56 is multiplied by 0.0427498, which is its weight (the cosine distances of eine term vectors to the average vector), 0.89 by 0.656847, etc.[22] We can see that 'eine' (typically a term that would appear in every stopword list for German) receives a very low weight, so it does not contribute much to the components of the sentence vector.

Step 8. Normalise sentence vectors to magnitude = 1.

At this point, both terms and sentences are modelled as vectors in the same space and can be compared for similarity when queried.

Now we can see how Queries work. When any string of text is queried:

i. Ariadne computes the vector representation for the query based on the words of the query (weighted sum, same as Step 7); a new vector is created (query

[21] The output of this step is https://github.com/ariannabetti/eIdeas/blob/master/vector

[22] Note also that the weights in the column **weight** are the real model's weight (to be found in the vocab file, see *Steps* 5-6), but the component values (c_1, c_2, c_3) above are fictional for the sake of the example, as c_1, c_2, c_3 are placeholders for co-occurring words.

vector) that represents the query by taking the weighted sum of the vectors of the words in the query.

ii. Ariadne computes the similarity between the query and all the sentences in the corpus, i.e. it computes which sentences have the highest cosine similarity to the query vector. Then, Ariadne ranks the results, making it possible to see which sentence vector has the highest (cosine) similarity to the query vector.

C. A second use-case: Bolzano's notion of essence

In this section we offer a control use case on Bolzano's concept of essence for the results of the use case on genus and differentia presented in the body of Ginammi et al. (2020). The aim of this section is to give evidence that it is not by accident that the use of BolVis/Ariadne generates valuable results for historico-philosophical investigations.

C.1 Bolzano's notion of essence

While close-reading Section §559 of Bolzano's *Wissenschaftslehre* (1837) (henceforth WL§559) following Query 1 in the main text of Ginammi et al. 2020, we learned that Bolzano distinguishes between essential ("wesentliche") and derivative ("abgeleitete") properties ("Beschaffenheiten") of an object. We hypothesize that this might be important, because we know from de Jong 1995 that for Kant the notion of essence ("Wesen") played an important role with regard to the question of (the possibility of) necessary predication or a priori knowledge, and with regard to his famous analytic-synthetic distinction. Both Bolzano's conception of necessary predication (necessary truth; Rusnock 2012, Siebel 1997, Textor 2013) and his analytic-synthetic distinction are regarded with perplexity by modern interpreters (Rusnock 2013, Siebel 2011; cf. de Jong 2001); if Bolzano has a notion of essence which plays the role it does in Kant, then this circumstance is prone to clarify Bolzano's views on necessary truth and the analytic-synthetic distinction. We pursue this as a second line of computational research for control, and aim to find the answer to:

(Q2) Does Bolzano have a notion of 'essence' similar to Kant's?

(Q2) is related to (Q) from the main text, but rests on different evidence (query terms) and therefore serves for control as far as the qualitative performance of BolVis/Ariadne is concerned. As said, we did not evaluate the tool on the basis

of a proper ground truth, because such a ground truth for our corpus does not yet exist.

We will discuss Kant's notion of essence in some more detail and then report the results from our research with BolVis aiming at answering (Q2).

C.2 Context and background: Kant

We have seen in section 3.1 of Ginammi et al. 2020 that - according to de Jong 1995, which we will follow also here - in Kant's view the proximate parts of a complex concept, which he also calls the constitutiva, rationes, or essentialia of that concept, make up the logical essence of that concept (de Jong 1995: 633, 635-7) and are always two: a kind or genus concept and a specific difference (differentia specifica) concept (de Jong 1995: 624, 633). In Kant's view, from the logical essence of a concept other characteristics of its objects follow; these characteristics he calls the object's attributes or propria (de Jong 1995: 633). There are attributes of two kinds: first, attributes which are contained in the concept as its remote parts, i.e. parts of the logical essence (so: parts of the genus or differentia), and which are called analytic attributes or analytic propria, and second, attributes which are not contained in the concept, but do follow from it as a consequence, which are called synthetic attributes or synthetic propria (de Jong 1995: 635-8).[23] Note that analytic and synthetic propria follow from the logical essence of a thing in different ways: an analytic proprium can be proven to follow from the logical essence on the basis of the principle of non-contradiction; a synthetic proprium can be proven to follow from the logical essence by means of some (other) kind of deduction (de Jong 1995: 633, 637-8).

The above underlies Kant's famous distinction between analytic and synthetic judgments: judgments in which the predicate is an essentiale or an analytic proprium of the subject, and thus is contained in the subject concept either as a proximate or as a remote part, Kant calls analytic judgments; judgments in which, instead, the predicate is a synthetic proprium of the subject, and thus is not contained in the subject concept but does follow from it, Kant calls synthetic judgments (de Jong 1995: 637). Furthermore, according to Kant, a thing's propria

[23] Note that Kant distinguishes between the proximate and remote parts of a concept, i.e. its logical essence (a complex of *genus-differentia* parts) and analytic propria (the parts of the *differentia*): this means that (complex) concepts with complex *differentiae* do have an internal order. We won't deal with the issue in this paper, but perhaps, seen in this light, Kant's views on concepts as mere conjunctions need a slightly more sophisticated treatment than is often argued in the secondary literature (e.g. Lapointe 2007, section 2; 2011, chapter 2).

and essentialia together (Kant calls this also the real or natural essence of that thing; de Jong 1995: 635, n. 50) make up all the necessary characteristics of that thing (de Jong 1995: 635-6). We thus see that in Kant's view analytic and synthetic judgments are two kinds of judgments which are a priori or necessarily true:

<table>
<tr><th>Type of knowledge</th><th>Corresponding properties</th><th>Consists of</th><th>Consists of</th><th>Predication results in</th></tr>
<tr><td rowspan="3">A priori, necessary</td><td rowspan="3">Real or natural essence</td><td rowspan="2">Logical essence</td><td>Genus*</td><td rowspan="2">Analytic judgment</td></tr>
<tr><td>Differentia*</td></tr>
<tr><td rowspan="2">Propria or attributes</td><td>Analytic propria **</td></tr>
<tr><td>Synthetic propria **</td><td rowspan="2">Synthetic judgment</td></tr>
<tr><td>A posteriori, contingent</td><td colspan="3">Accident</td></tr>
<tr><td colspan="5">* The parts of the logical essence are also called by Kant the constitutiva, rationes, or essentialia of that concept</td></tr>
<tr><td colspan="5">** Analytic propria are contained in the logical essence as parts; synthetic propria follow as a consequence from the logical essence.</td></tr>
</table>

Table 1: Our reconstruction of Kant's Theory of Concepts from de Jong 1995

We now want to know: how are things in Bolzano? In other words, to what extent does this table represent Bolzano's views?

C3. Results

A natural first step to take is to query for the term "essence".

Query 1. A query for "Wesen" (essence) does not give the desired result: almost all sentences that BolVis returns are about the infinite being ("das unendlichen Wesen"), i.e. God (Bolzano was also a priest and theologian). We try essential property ("wesentliche Beschaffenheit"), which works better, and the first four passages we obtain are from WL§111, a section dedicated to the distinction between essential and extra-essential property concepts ("wesentliche und außerwesentliche Beschaffenheitsvorstellungen").

Close reading. We turn to the source text, WL§111, and learn that essential properties are for Bolzano those properties that an object has in virtue of falling under a certain concept, or in other words, properties that follow from the

concept; extra-essential properties are properties that do not follow from the concept and thus belong to the object contingently. Thus, like Kant, Bolzano calls properties which a priori can be attributed to an object, i.e. the object's necessary properties, essential properties. But why does Bolzano contrast essential properties to derivative properties in WL§559? It does not seem to be the case that the "derivative properties" of WL§559 are the same as the "extra-essential" properties of WL§111, first, because Bolzano does not write anything to this effect in WL§111 and §559, and second, because we know that derivability ("Ableitbarkeit") is Bolzano's notion of logical consequence, and thus refers to something a priori and necessary - not contingent. So: what exactly are "derivative" properties according to Bolzano?

Query ii. We query for "abgeleitete Beschaffenheit" (derivative property) and get many sentences from Bolzano's mathematical works in which abgeleitet is used in another sense, works which BolVis enables us to filter out, and then obtain the title of WL§113 "Original and derivative property concepts" ("Ursprüngliche und abgeleitete Beschaffenheitsvorstellungen").

Close reading. We read §113 and learn that original properties, which Bolzano also calls constitutive properties, are those properties of a thing which are a part of the concept under which that thing falls; all the other properties are derivative properties. It thus seems that Bolzano's constitutive properties correspond to Kant's constitutiva plus Kant's analytic propria, that is, all parts of a concept, proximate as well as remote. At this point, we expect that Bolzano's derivative properties correspond to Kant's synthetic propria, that is, the properties that are not contained in a concept but that follow from it in some other way. We need confirmation that Bolzano's derivative properties do indeed follow from the concept under which the object falls, or in other words, that the essential properties comprise not only the original properties (Bolzano confirms this explicitly in WL§113), but also the derivative properties (Bolzano writes nothing to this effect in WL§113).

Query iii. Unfortunately, queries for terms related to "derivative property" do not give relevant results - possibly because the results are dominated by sentences from the mathematical works (only 2 out of the 50 results are from non-mathematical works).

Close reading. We turn again to the section in which Bolzano distinguishes between essential and derivative properties of an object (WL§559) and follow his own cross-references, which bring us via WL§515 to WL§502. Here we find what we had been looking for: Bolzano distinguishes between two senses of essence, a broad and a narrow one. The difference between the two Bolzano formulates on

the basis of his distinction between grounding and derivability: the essence in the strict sense, which he calls also fundamental essence ("Grundwesen"), comprises those properties that follow from that concept as a consequence from its ground, whereas the essence in the broad sense ("Wesen") comprises also those properties that can be derived from the concept. For example, Bolzano writes, the property of being a system of three points is (or belongs to) the fundamental essence of a triangle (Grundwesen); the property of having angles which sum up to two right ones is a merely derivative property (and thus belongs to the essence in the broad sense - Wesen -, but not to the fundamental essence - Grundwesen) of triangle, because the property of having angles which sum up to two right ones is grounded in the property of being a system of three points, and thus follows from, but is not part of, the concept of triangle itself. Bolzano writes here explicitly that derivative properties belong to the essence in the broad sense, and thus we conclude that indeed, Bolzano's derivative properties correspond to Kant's synthetic propria.

We are now in a position to answer:

(Q2) Does Bolzano have a notion of "essence" similar to Kant's?

Yes, Bolzano has a notion of "essence" similar to Kant's. For both Kant and Bolzano, in necessary statements the predicate belongs to the essence of the subject. Furthermore, both Kant and Bolzano distinguish a broad and a narrow sense of "essence"; their respective broad and narrow senses of essence are however slightly different. For Kant, the logical essence of an object comprises only the proximate parts of the object's concept, the constitutiva: the genus proximum and the differentia specifica; all other necessary properties, i.e. the propria belong to the real or natural essence. In turn, the propria divide for Kant into analytic propria, i.e. those properties that are contained in the concept as remote parts, and synthetic propria, i.e. those properties that follow from, but are not contained in the concept. For Bolzano, the fundamental essence comprises all properties that are part of the concept (unlike Kant, Bolzano does not distinguish between the proximate and the remote parts of a concept), whereas the essence in the broad sense comprises also all properties that follow from, but are not contained in the concept.

The main difference between Kant and Bolzano is thus that for Kant, the essence in the strict sense, or logical essence, comprises only the constitutiva (i.e. the proximate parts of the concept), whereas for Bolzano, the essence in the strict sense, or fundamental essence, comprises all properties that are parts of the

concept (both its proximate and remote parts), that is, Kant's constitutiva plus analytic propria. Note that this makes sense in the light of the results reported in the main text: constitutiva play for Kant a special role which they do not play for Bolzano. Namely, as we have seen in section 3.1 of the main text, for Kant the constitutiva (i.e. the genus and differentia) determine the concept's position within the conceptual hierarchy (where the conceptual hierarchy for Kant is based on the concepts' intension - and, in virtue of the canon of reciprocity, on extension, too), whereas, as we discussed in section 5 of the main text, this is not the case for Bolzano: for Bolzano (who rejects the canon of reciprocity), a concept's place in the conceptual hierarchy is determined by its extension only.

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